Quiz 4

Name: key Row:______

[6] **1.** Differentiate the function $y = \ln(e^{(x^2)} + 1)$.

$y = \ln u$ where $u = e^{(x^2)} + 1$

$u = e^{w} + 1$ where $w = x^2$

1 1 1

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} = \frac{dy}{du}\cdot\frac{du}{dw}\cdot\frac{dw}{dx} = \frac{1}{u}\cdot(e^{w}+0)\cdot(2x)$$

1 1

$$= \boxed{\frac{1}{e^{(x^2)}+1}\cdot(e^{(x^2)})\cdot 2x}$$ 1

2. Evaluate the integrals. Simplify your answers as much as possible.

[7] **a.** $$\int_1^e \frac{\sqrt{\ln x}}{x}\,dx = \int_{\ln 1}^{\ln e} \sqrt{u}\,du = \int_{\ln 1}^{\ln e} u^{1/2}\,du = \left[\frac{2}{3}u^{3/2}\right]_{\ln 1}^{\ln e}$$

1 1

1 $u = \ln x$

1 $du = \frac{1}{x}\,dx$

1 $x = 1 \to u = \ln 1$

$x = e \to u = \ln e$

$$= \frac{2}{3}\left[(\ln e)^{3/2} - (\ln 1)^{3/2}\right]$$ 1

$$= \frac{2}{3}\left[1^{3/2} - 0^{3/2}\right] = \boxed{\frac{2}{3}}.$$ 1

[7] **b.** $$\int_0^\pi \frac{e^x + \cos x}{e^x + \sin x}\,dx = \int_{e^0+\sin 0}^{e^\pi+\sin\pi} \frac{du}{u} = \Big[\ln u\Big]_{e^0+\sin 0}^{e^\pi+\sin\pi}$$

1 1

1 $u = e^x + \sin x$

1 $du = (e^x + \cos x)\,dx$

1 $x = 0 \to u = e^0 + \sin 0$

$x = \pi \to u = e^\pi + \sin\pi$

$$= \ln(e^\pi + \sin\pi) - \ln(e^0 + \sin 0)$$

1

$$= \ln(e^\pi + 0) - \ln(1 + 0)$$

$$= \pi\cdot\ln e - \ln 1$$

$$= \pi\cdot 1 - 0$$

$$= \boxed{\pi}$$ 1