

(a) The collection $M_{n,n}$ of all $n \times n$ matrices may be regarded as the collection $\mathbf{R}^{n^2} = \mathbf{R}^n \times \cdots \times \mathbf{R}^n$ of all n -tuples of n -vectors by writing a matrix $A = (\mathbf{a}_1, \dots, \mathbf{a}_n)$ in terms of its columns \mathbf{a}_i . Notice that vector addition and scalar multiplication coincide in $M_{n,n}$ and $\mathbf{R}^n \times \cdots \times \mathbf{R}^n$. Prove that $\det : \mathbf{R}^n \times \cdots \times \mathbf{R}^n \rightarrow \mathbf{R}$ is multilinear.

(b) Show that \det is differentiable, and compute its derivative.