

Suppose  $\mathbf{f} : \mathbb{R}^n \rightarrow \mathbb{R}^m$  satisfies  $\mathbf{f}(t\mathbf{a}) = t\mathbf{f}(\mathbf{a})$  for all  $\mathbf{a} \in \mathbb{R}^n$  and  $t \in \mathbb{R}$ . Prove that if  $\mathbf{f}$  is differentiable at the origin, then  $\mathbf{f}$  is linear. *Hint:* It is enough to show that  $\mathbf{f} = D\mathbf{f}(\mathbf{0})$ .