An $n \times n$ matrix A is said to be *orthogonal* if AA^T equals the identity matrix I_n.

(a) Prove that the collection $O(n)$ of all orthogonal $n \times n$ matrices is a group; i.e., the product of two orthogonal matrices is again orthogonal.

(b) Let I be an open interval containing 0, $\mathbf{c} : I \to O(n)$ a differentiable curve with $\mathbf{c}(0) = I_n$. Prove that the matrix $A = \mathbf{c}'(0)$ is skew-symmetric; i.e., $A + A^T = 0$. *Hint:* If $M_{n,n} \cong R^{n^2}$ denotes the vector space of all $n \times n$ matrices, then the multiplication map

$$\begin{aligned} m : M_{n,n} \times M_{n,n} &\to M_{n,n} \\ (A, B) &\mapsto A \cdot B \end{aligned}$$

is bilinear, so that by a theorem proved in class, m is differentiable with derivative

$$Dm(A, B)(C, D) = m(A, D) + m(C, B) = A \cdot D + C \cdot B.$$