Exam II

Topology (Math 5853)

November 20, 2013

Choose four problems. If you do five, please say which four you want graded.

1(a) Show that every closed subspace of a compact space is compact.
(b) Show that every compact Hausdorff space is regular.

2. Let $p\colon X \to Y$ be a quotient map. Show that if each set $p^{-1}(\{y\})$ is connected and Y is connected, then X is connected.

3(a) Suppose X is locally compact and Hausdorff, but not compact. Define the topology for the one-point compactification $X \cup \{\infty\}$. (That is, name all the open sets.)
(b) Give an example of two non-homeomorphic locally compact Hausdorff spaces whose one-point compactifications are homeomorphic.

4. Give the details of the argument showing that if X and Y are connected then so is $X \times Y$. State carefully any results that you use.

5(a) Give the definitions of *components* and *path components* of a space X.
(b) What is the precise relationship between components and path components of a space? Give proofs and/or counterexamples for your assertions.