Exam I

Topology (Math 5853)

October 14, 2013

---

**1(a)** State the axioms for $\mathscr{B}$ to be a basis.
**(b)** Define the topology $\mathscr{T}$ generated by $\mathscr{B}$.
**(c)** Suppose $\mathscr{B}_1$ and $\mathscr{B}_2$ are bases generating the topologies $\mathscr{T}_1$ and $\mathscr{T}_2$ respectively on a set $X$. State a necessary and sufficient criterion in terms of $\mathscr{B}_1$ and $\mathscr{B}_2$ for $\mathscr{T}_1$ to be finer than $\mathscr{T}_2$.

---

**2(a)** Say what it means for an ordered set $A$ to be *well-ordered*.
**(b)** Define the *dictionary ordering* on $A \times B$, where $A$ and $B$ are ordered sets.
**(c)** Show that if $A$ and $B$ are well-ordered, then so is $A \times B$ (in the dictionary order).

---

**3.** Let $f\colon \mathbb{R} \to \mathbb{R}^\omega$ be given by $f(t) = (t, \frac{1}{2}t, \frac{1}{4}t, \frac{1}{8}t, \ldots)$. Show that $f$ is not continuous if $\mathbb{R}^\omega$ is given the box topology.

---

**4.** Let $Y$ be a Hausdorff space. Suppose $g, h\colon X \to Y$ are continuous maps. Prove that the set $\{x \in X \mid g(x) = h(x)\}$ is closed. [Hint: use $Y \times Y$.]

---

**5.** Let $X = \mathbb{Z} \times [0, 1]$ and define an equivalence relation $\sim$ on $X$ by: $(n, 1) \sim (n+1, 0)$ for all $n \in \mathbb{Z}$ (no other identifications are made).
**(a)** Draw a picture of $X$ and also indicate what the quotient space $X^*$ looks like.
**(b)** Show that there is a continuous bijection $X^* \to \mathbb{R}$. State carefully what needs to be verified in order to define this function and know that it is continuous. Verify that the required properties hold.