Final Exam
Math 4513–002 Capstone
Due May 6, 2013

Rules. You may refer to your class notes, homework and quizzes, and reference items linked to from the course web page. You may not use other sources (books, online resources, etc) or discuss the exam with others. Please write out your answers neatly and in complete sentences. Please feel free to ask me for a hint if you are stuck. The exam is due at the end of the final exam period, 3:30 pm on May 6. You can turn it in to me at my office, or to my mailbox on the fourth floor (talk to the receptionist there).

---

1. Determine which of the 15-puzzle configurations below can be solved and which are impossible to solve. (Explain.)

12	1	8	3
5	14	4	13
9		7	2
6	11	15	10

14	6	8	11
2	9	10	5
12	13	1	4
15	3	7	

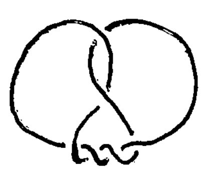
	6	11	15
5	3	9	1
10	4	12	13
7	8	14	2

---

2. Write the transposition $(3\ 7)$ as a product of *consecutive* transpositions (ie. of the form $(k\ k{+}1)$).

---

3. Determine which of the two knots below are tri-colorable. (Explain, or give a tri-coloring.)



---

4. Show that the rational tangle $2\ 1\ a_1\ a_2\ \cdots\ a_n$ is equivalent to the rational tangle $-2\ 2\ a_1\ a_2\ \cdots\ a_n$ both by using continued fractions and by drawing a picture.

---

5. Show that every generic closed curve in the plane can be made into an *alternating* knot diagram, as follows. *Generic* means that there are only finitely many intersection points, and these each involve only two strands at a time, crossing at some non-zero angle (ie. tangencies are not allowed).

(a) Show that if the regions formed by the curve admit a checkerboard coloring, then there is a way to assign crossings that make the diagram alternating.

(b)* Show that the regions always admit a checkerboard coloring. [Hint: curves with self-intersections can be turned into the circle using Reidemeister moves I–III. Show that these moves preserve the property that a checkerboard coloring exists. Now what?]