Name: **Extra Credit Quiz 1 — Math 3113-005**

1. Find the partial fraction expansion of $\frac{1}{s(s^2+1)}$ and then find $\mathcal{L}^{-1}\left\{\frac{1}{s(s^2+1)}\right\}$.

$$\frac{1}{s(s^2+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+1}, \quad 1 = A(s^2+1) + s(Bs+C)$$

$\underline{s=0}$ $\quad 1 = A$

$\underline{s=i}$ $\quad 1 = -B + Ci$

$B = -1, \ C = 0 \qquad$ so $\frac{1}{s(s^2+1)} = \frac{1}{s} - \frac{s}{s^2+1}$

$$\mathcal{L}^{-1}\left\{\frac{1}{s(s^2+1)}\right\} = 1 - \cos t.$$

2. Use the rule $\mathcal{L}\left\{\int_0^t f(\tau)d\tau\right\} = \frac{F(s)}{s}$ to find $\mathcal{L}^{-1}\left\{\frac{1}{s(s^2+1)}\right\}$. (Do the integral carefully.)

Let $F(s) = \frac{1}{s^2+1}$, so $f(t) = \sin t$.

Then $\mathcal{L}^{-1}\left\{\frac{1}{s(s^2+1)}\right\} = \mathcal{L}^{-1}\left\{\frac{F(s)}{s}\right\} = \int_0^t \sin\tau \, d\tau$

$$= \left[-\cos\tau\right]_0^t = -\cos t + \cos(0)$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s(s^2+1)}\right\} = 1 - \cos t.$$

3. Set up the following initial value problem as an equation involving $X(s) = \mathcal{L}\{x(t)\}$.

$2x''(t) + 6x'(t) + x(t) = \cos(12t), \ \ x(0) = -5, \ \ x'(0) = -3$

$\mathcal{L}\{x\} = X(s), \quad \mathcal{L}\{x'\} = sX(s) + 5, \quad \mathcal{L}\{x''\} = s^2X(s) + 5s + 3.$

$$2\left(s^2X(s) + 5s + 3\right) + 6\left(sX(s) + 5\right) + X(s) = \frac{s}{s^2+144}.$$