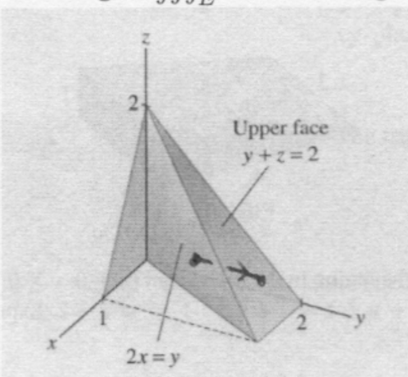
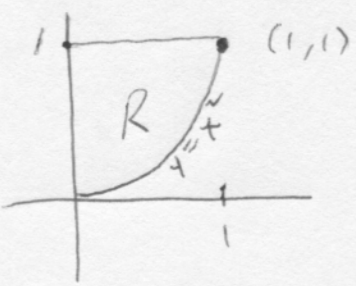
1(a) The figure below shows the region E bounded by $y+z=2$, $2x=y$, $x=0$, and $z=0$. Set up the integral $\iiint_E xe^x\,dV$ using the order $dV = dydzdx$. *Do not* evaluate the integral.



$$\boxed{\int_0^1 \int_0^{2-2x} \int_{2x}^{2-z} xe^x\,dy\,dz\,dx}$$

1(b) Sketch the region, and evaluate the integral $\int_0^1 \int_{x^2}^1 x^3 \sin(y^3)\,dydx$ by reversing the order of integration.



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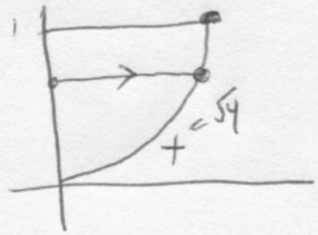


$$\begin{cases} x^2 \le y \le 1 \\ 0 \le x \le 1 \end{cases}$$

$$\int_0^1 \int_0^{\sqrt{y}} x^3 \sin(y^3)\,dx\,dy$$

$$= \int_0^1 \frac{1}{4}x^4 \sin(y^3)\Big|_0^{\sqrt{y}}\,dx = \int_0^1 \frac{1}{4}y^2\sin(y^3)\,dy$$

$u = y^3$, $du = 3y^2dy$

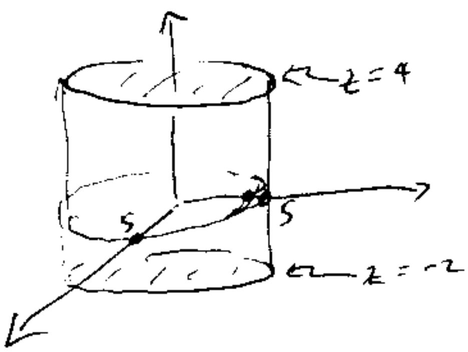
$$= \int_0^1 \frac{1}{12}\sin(u)du = \frac{-\cos(u)}{12}\Big|_0^1$$

$$= \boxed{\frac{1}{12}\left(-\cos(1) + \cos(0)\right)}$$

2(a) Describe carefully in words the following surfaces (given with respect to spherical coordinates):

- $\theta = \pi$ → the vertical half-plane starting along the z-axis and extending along the negative x-axis
- $\rho = 5$ → the sphere of radius 5 centered at the origin
- $\phi = \pi/2$ → the xy-plane

2(b) Use cylindrical coordinates to evaluate $\iiint_E \sqrt{x^2+y^2}\,dV$, where E is the region that lies inside the cylinder $x^2+y^2=25$ and between the planes $z=-2$ and $z=4$.



$$\sqrt{x^2+y^2} = r$$
$$dV = r\,dz\,dr\,d\theta$$

$$\int_0^{2\pi}\int_0^5\int_{-2}^4 r^2\,dz\,dr\,d\theta$$

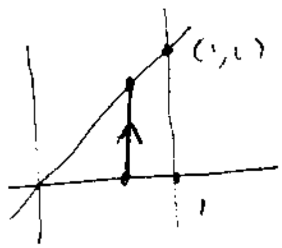
$$= \int_0^{2\pi} d\theta \int_0^5 r^2\,dr \int_{-2}^4 dz = 12\pi \cdot \frac{1}{3} r^3 \Big|_0^5$$

$$= 4\pi \cdot 125$$

$$= \boxed{500\pi}$$

3(a) Evaluate the integral $\iint_R e^{x^2}\, dA$, where R is the region bounded by the lines $y = 0$, $x = y$, and $x = 1$. [Warning: you must choose the correct order of integration for it to work.]

Can't integrate e^{x^2} directly, so integrate in y first.



$$\int_0^1 \int_0^x e^{x^2}\, dy\, dx = \int_0^1 y e^{x^2} \Big|_0^x dx$$

$$= \int_0^1 x e^{x^2} dx = \frac{1}{2}\int_0^1 e^u\, du \quad \left[\begin{array}{l} u = x^2 \\ du = 2x\, dx \end{array}\right]$$

$$= \frac{1}{2} e^u \Big|_0^1 = \boxed{\frac{1}{2}(e-1)}$$

3(b) A curve C is parametrized as $\mathbf{r}(t) = \langle \cos t, 2e^t\rangle$ for $0 \le t \le \pi$. Express each of the following as ordinary integrals in t. *Do not* evaluate the integrals.

- $\int_C x^2 y\, ds$
- $\int_C x^2 y\, dx$
- $\int_C \langle x^2, y\rangle \cdot d\mathbf{r}$

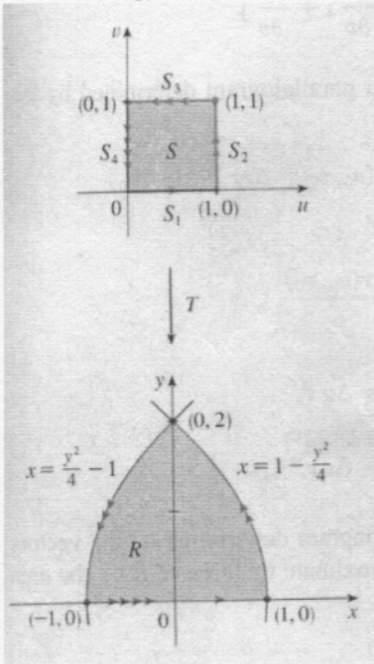
$$r'(t) = \langle \underbrace{-\sin t}_{x'(t)}, \underbrace{2e^t}_{y'(t)} \rangle$$

$$ds = |r'(t)|\, dt = \sqrt{\sin^2 + 4e^{2t}}\, dt.$$

- $\int_C x^2 y\, ds = \boxed{\int_0^\pi \cos^2 t \cdot 2e^t \sqrt{\sin^2 t + 4e^{2t}}\, dt}$
- $\int_C x^2 y\, dx = \boxed{\int_0^\pi \cos^2 t \cdot 2e^t(-\sin t)\, dt}$
- $\int_C \langle x^2, y\rangle \cdot d\vec{r} = \int_0^\pi \langle \cos^2 t, 2e^t\rangle \cdot \langle -\sin t, 2e^t\rangle\, dt$

$$= \boxed{\int_0^\pi (-\cos^2 t \sin t + 4e^{2t})\, dt}$$

4. Let R be the region bounded by the x-axis and the parabolas $x = 1 - \frac{y^2}{4}$ and $x = \frac{y^2}{4} - 1$, as shown below. The transformation $x = u^2 - v^2$, $y = 2uv$ takes the unit square S in the uv-plane to the region R. **(a)** Find the Jacobian of this transformation. **(b)** Use the Jacobian to convert the integral $\iint_R x\,dA$ into an integral over S in the variables u, v. **(c)** Evaluate the integral.



(a) $$J = \left| \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\frac{\partial y}{\partial u} \right|$$

$$= \left| 2u \cdot 2u - 2v(-2v) \right|$$

$$= \boxed{4u^2 + 4v^2}$$

(b) $$\iint_R x\,dA = \iint_S x(u,v)\, J\, dA$$

$$= \boxed{\int_0^1 \int_0^1 (u^2 - v^2)(4u^2 + 4v^2)\, du\, dv}$$

(c) $$= \int_0^1 \int_0^1 4(u^4 - v^4)\, du\, dv$$

$$= 4\int_0^1 \left(\tfrac{1}{5}u^5 - uv^4 \Big|_0^1 \right) dv = 4\int_0^1 \left(\tfrac{1}{5} - v^4 \right) dv$$

$$= 4\left[\tfrac{1}{5}v - \tfrac{1}{5}v^5 \right]_0^1 = 4\left(\tfrac{1}{5} - \tfrac{1}{5} \right)$$

$$= \boxed{0}$$