1(a) Find the values of x for which the vectors $\langle 4, x, 3\rangle$ and $\langle 2x, x, 5\rangle$ are orthogonal.

$$\langle 4, x, 3\rangle \cdot \langle 2x, x, 5\rangle = 0$$

$$8x + x^2 + 15 = 0$$

$$(x+3)(x+5) = 0$$

$$\boxed{x = -3, \quad x = -5}$$

1(b) A parallelepiped has one corner at the origin and three adjacent corners at the points $(2, 1, 2)$, $(3, 3, 0)$ and $(1, 1, 1)$. Find the volume of the parallelepiped.

Sides are given by the vectors $\langle 2,1,2\rangle$, $\langle 3,3,0\rangle$, and $\langle 1,1,1\rangle$. The volume is given by the scalar triple product (in absolute value):

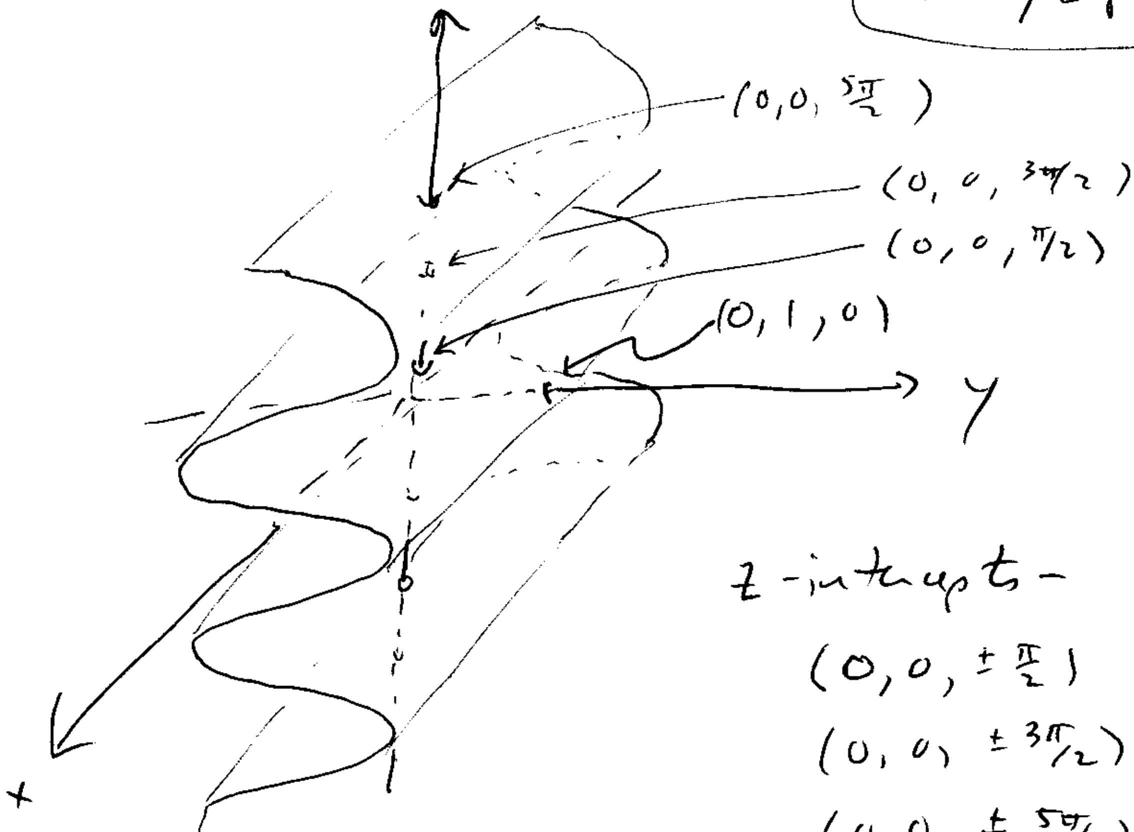
$$V = \left| \langle 2,1,2\rangle \cdot \left(\langle 3,3,0\rangle \times \langle 1,1,1\rangle \right) \right|$$

$$= \left| \langle 2,1,2\rangle \cdot \langle 3,-3,0\rangle \right|$$

$$= \left| 6 - 3 + 0 \right| = \boxed{3}.$$

2. Describe and sketch the surface $y = \cos z$. Draw it as carefully as you can, and include any noteworthy details (intercepts, asymptotes, etc).

Cylinder over curve $y = \cos z$ in yz plane.



z-intercepts –

$(0, 0, \pm \frac{\pi}{2})$

$(0, 0, \pm 3\pi/2)$

$(0, 0, \pm 5\pi/2)$

$\vdots$

y-intercept $(0, 1, 0)$

no x-intercept

3. Consider the two planes $x + y + z = 1$ and $6x + 4y - 2z = 0$.

(a) Find the direction of the line of intersection of the two planes.

(c) Find the cosine of the angle between the two planes.

(a) The line of intersection is orthogonal to the normal vectors of both planes.

$\vec{n}_1 = \langle 1,1,1 \rangle$ and $\vec{n}_2 = \langle 6,4,-2 \rangle$.

An orthogonal vector is given by

$$\vec{n}_1 \times \vec{n}_2 = \langle 1,1,1 \rangle \times \langle 6,4,-2 \rangle$$

$$= \langle -2-4,\ 6-(-2),\ 4-6 \rangle$$

$$= \boxed{\langle -6, 8, -2 \rangle}$$

(c) This angle is the angle between the normal vectors.

We have $\vec{n}_1 \cdot \vec{n}_2 = |\vec{n}_1||\vec{n}_2| \cos\theta$

So

$$6+4-2 = \sqrt{3}\sqrt{36+16+4} \cdot \cos\theta$$

$$8 = \sqrt{3 \cdot 56} \cos\theta$$

$$\boxed{\cos\theta = \frac{8}{\sqrt{168}}}$$

4. A curve is given by parametric equations $\mathbf{r}(t) = \langle \cos t, 3t, 2\sin 2t\rangle$.
(a) Find $\mathbf{r}'(t)$.
(b) Find the unit tangent vector $\mathbf{T}(t)$ at the point where $t = 0$.
(c) Find parametric equations for the tangent line to the curve at the point $(1, 0, 0)$.

(a) $\vec{r}\,'(t) = \langle -\sin t, 3, 4\cos 2t \rangle$

(b)
$$\vec{T}(0) = \frac{\vec{r}\,'(0)}{|\vec{r}\,'(0)|} = \frac{\langle 0, 3, 4\rangle}{|\langle 0, 3, 4\rangle|} = \frac{1}{\sqrt{9+16}}\langle 0, 3, 4\rangle$$
$$\boxed{= \langle 0, \tfrac{3}{5}, \tfrac{4}{5}\rangle}$$

(c) note that $(1,0,0)$ is the position of $\vec{r}(t)$ at $t=0$. So the tangent line has direction $\langle 0, \frac{3}{5}, \frac{4}{5}\rangle$.

Parametric equations are
$$\vec{l}(t) = \langle 1, 0, 0\rangle + t\langle 0, \tfrac{3}{5}, \tfrac{4}{5}\rangle$$
$$\boxed{\vec{l}(t) = \langle 1, \tfrac{3}{5}t, \tfrac{4}{5}t \rangle}.$$

or
$$\begin{cases} x(t) = 1 \\ y(t) = 3/5\, t \\ z(t) = 4/5\, t \end{cases}$$