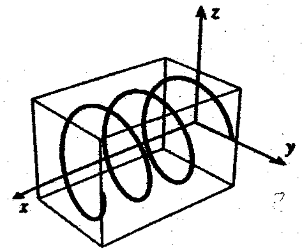
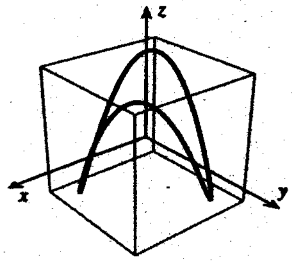
1. Match the graphs with the vector valued functions; give brief explanations.

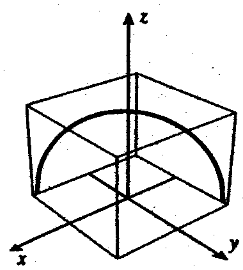
(A) $\mathbf{r}(t) = \langle t, -t, \sqrt{2 - t^2} \rangle$
(B) $\mathbf{r}(t) = \langle \sin \pi t, -t, t \rangle$
(C) $\mathbf{r}(t) = \langle \sin t, \cos t, \sin 2t \rangle$
(D) $\mathbf{r}(t) = \langle \frac{1}{2}t, \cos 3t, \sin 3t \rangle$



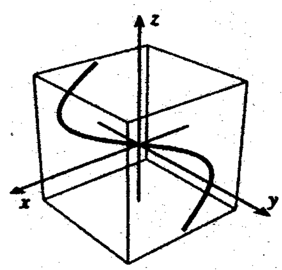
I



II



III



IV

A ↔ III curve lies within the plane $y = -x$; z coord. is highest when $t = 0$ (and $x = y = 0$); semi-circular shape looks right for $z = \sqrt{2 - t^2}$

B ↔ IV lies within plane $y = -z$; oscillation in x-direction only

C ↔ II circular xy-motion, combined with vertical oscillation, twice as fast as horizontal oscillations

D ↔ I circular yz-motion, combined with steady motion in x-direction

2(a) Find the cosine of the angle between the planes $x + y + z = 1$ and $x - 2y + 3z = 1$.

normal vectors are $\underline{n}_1 = \langle 1, 1, 1 \rangle$ and $\underline{n}_2 = \langle 1, -2, 3 \rangle$.

If $\theta = \text{angle}(\underline{n}_1, \underline{n}_2)$ then

$$\cos\theta = \frac{\underline{n}_1 \cdot \underline{n}_2}{|\underline{n}_1||\underline{n}_2|} = \frac{1-2+3}{\sqrt{3}\sqrt{14}} = \frac{2}{\sqrt{42}}.$$

Note since $\cos\theta \geq 0$, this angle is acute, so θ is also the angle between the planes.

$$\boxed{\cos\theta = \frac{2}{\sqrt{42}}}$$

(b) Find $\text{comp}_{\mathbf{a}}\mathbf{b}$ and $\text{proj}_{\mathbf{a}}\mathbf{b}$ for $\mathbf{a} = 3\mathbf{i} + 6\mathbf{j} - 2\mathbf{k}$ and $\mathbf{b} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$.

$$\text{comp}_{\underline{a}}\underline{b} = \frac{\underline{a} \cdot \underline{b}}{|\underline{a}|} = \frac{3+12-6}{\sqrt{9+36+4}} = \boxed{\frac{9}{7}}$$

$$\text{proj}_{\underline{a}}\underline{b} = \frac{\underline{a} \cdot \underline{b}}{|\underline{a}|^2}\underline{a} = \frac{9}{49}\langle 3, 6, -2 \rangle$$

$$= \boxed{\left\langle \frac{27}{49}, \frac{54}{49}, \frac{-18}{49} \right\rangle}$$

3. Consider the parallelepiped with adjacent edges

$$\mathbf{u} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$$
$$\mathbf{v} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$$
$$\mathbf{w} = \mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$$

(a) Find the volume.
(b) Find the area of the face determined by **u** and **w**.
(c) Find two *unit* vectors that are normal to the face determined by **u** and **w**.
(d) Which of your normal vectors points into the parallelepiped, and which points away from it? How can you decide for sure?

(a) volume $= |\underline{u} \cdot (\underline{v} \times \underline{w})| = |\underline{u} \cdot \langle -3, -1, 2 \rangle|$

$= |-9 - 2 + 2| = \boxed{9}$

(b) area of face $= |\underline{u} \times \underline{w}|$

$= |\langle 3, -8, 7 \rangle| = \sqrt{9 + 64 + 49} = \boxed{\sqrt{122}}$

(c) unit vectors are

$\boxed{\underline{n}_1 = \left\langle \frac{3}{\sqrt{122}}, \frac{-8}{\sqrt{122}}, \frac{7}{\sqrt{122}} \right\rangle}$ and $\boxed{\underline{n}_2 = \left\langle \frac{-3}{\sqrt{122}}, \frac{8}{\sqrt{122}}, \frac{-7}{\sqrt{122}} \right\rangle}$.

(d) equivalently, which $\underline{n}$ is on the same side / opposite side of the plane spanned by $\underline{u}, \underline{w}$ as $\underline{v}$?

Compare angle between $\underline{n}$ and $\underline{v}$.

$\underline{n}_1 \cdot \underline{v} = \frac{3}{\sqrt{122}} + \frac{-8}{\sqrt{122}} + \frac{14}{\sqrt{122}} = \frac{9}{\sqrt{122}} > 0$, acute angle

so $\boxed{\underline{n}_1 \text{ points in}}$.

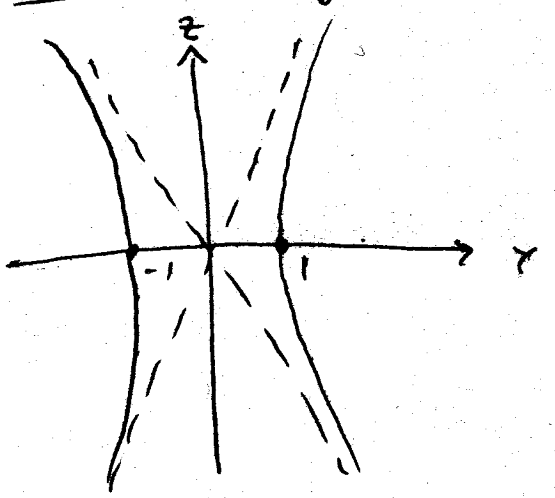
$\underline{n}_2 \cdot \underline{v} = \frac{-3}{\sqrt{122}} + \frac{8}{\sqrt{122}} + \frac{-14}{\sqrt{122}} = \frac{-9}{\sqrt{122}} < 0$, obtuse angle

so $\boxed{\underline{n}_2 \text{ points out}}$.

4. For the quadric surface $-x^2 + 4y^2 - z^2 = 4$, draw its traces in the planes $x = 0$ and $z = 0$. Which kind of quadric surface is this? Draw a picture of the surface.

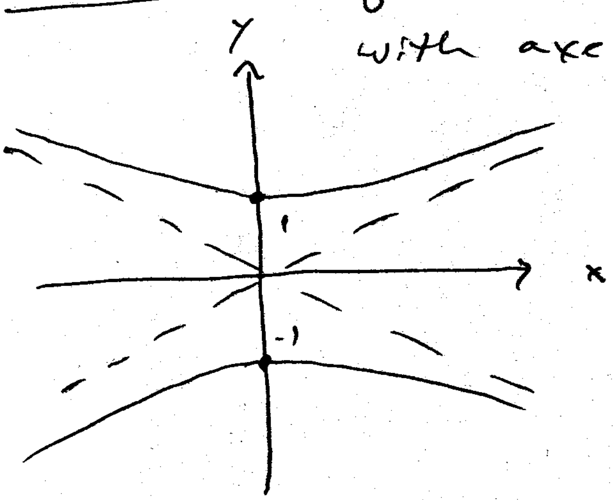
<u>trace $x=0$</u> get $4y^2 - z^2 = 4$ in yz plane; a hyperbola

asymptotes $4y^2 = z^2$, i.e. $y = \pm \frac{z}{2}$

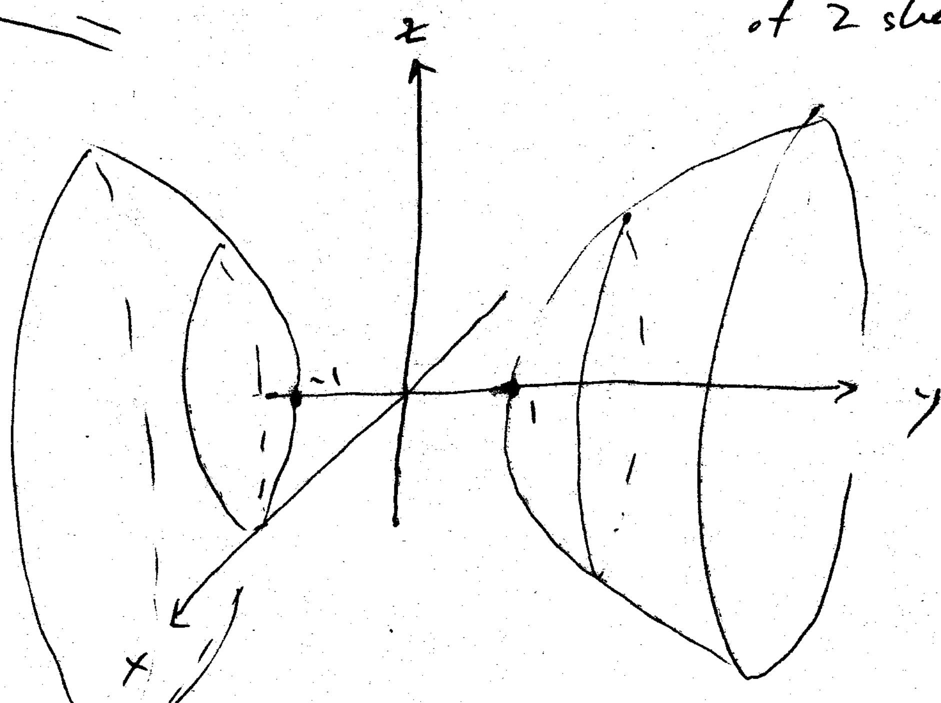


<u>trace $z=0$</u> get $-x^2 + 4y^2 = 4$ in xy-plane; hyperbola

with axes $y = \pm \frac{x}{2}$



<u>surface</u> hyperboloid of 2 sheets!



- centered along y-axis
- does not meet the xz-plane