Name: ID #:

Exam III

Math 2433-010

November 23, 2010

Problem 1: Problem 2:

Problem 3: Problem 4:

Problem 5:

Total:

1(a) Find the values of x such that the vectors $\langle 4, x, 4\rangle$ and $\langle 2x, x, 5\rangle$ are orthogonal.

(b) A parallelepiped has one corner at the origin and three adjacent corners at the points $(2, 1, 2)$, $(3, 3, 0)$ and $(1, 1, 1)$. Find the volume of the parallelepiped.

2(a) Find a power series expansion, centered at 0, for the function $f(x) = \frac{1}{x+5}$.
(b) What is the radius of convergence of this power series?
(c) What is $f^{(50)}(0)$?

3(a) Find two unit vectors that are orthogonal to both $3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $\mathbf{i} - 2\mathbf{k}$.

(b) A rectangular box has side lengths 1, 1, and 2. Find the cosine of the angle between the diagonal of the box and the diagonal of the long rectangular face (these diagonals meet at a corner). [Draw a picture!]

4(a) Write down the MacLaurin series for e^x.
(b) Write down $\frac{e^x}{x}$ as an infinite series.
(c) Write down $\int \frac{e^x}{x}\, dx$ as an infinite series.

5. Let $\mathbf{u}$ and $\mathbf{v}$ be the sides of a parallelogram, considered as vectors, with initial point at the same corner.

(a) Express the diagonals of the parallelogram as vectors.

(b) Use properties of the dot product to express the squared-lengths of the diagonals in terms of $|\mathbf{u}|$, $|\mathbf{v}|$, and $\mathbf{u} \cdot \mathbf{v}$.

(c) Show that if the lengths of the diagonals are equal, then $\mathbf{u} \cdot \mathbf{v} = 0$. What does this say about the parallelogram?