1. Show, using any valid method (accompanied by a brief explanation), that $(p \wedge q) \to r$ and $(p \to r) \wedge (q \to r)$ are not logically equivalent.

p	q	r	$p \wedge q$	$(p\wedge q)\to r$	$p\to r$	$q\to r$	$(p\to r)\wedge(q\to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	F	T	T	T	T
T	F	F	F	T	F	T	F
F	T	T	F	T	T	T	T
F	T	F	F	T	T	F	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

From the above truth-value table, we see that: if p is true, q and r are false, then $(p\wedge q)\to r$ is true, but $(p\to r)\wedge(q\to r)$ is false.

So, $(p\wedge q)\to r$ and $(p\to r)\wedge(q\to r)$ are <u>not</u> logically equivalent.

✓

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2(a) Translate each of these statements into an English statement that expresses a mathematical fact. The domain in each case consists of all real numbers.

(i) $\exists x \exists y((x^2 > y) \wedge (x < y))$

(ii) $\forall x((x \neq 0) \rightarrow \exists y(xy = 1))$

(i) There is a real number x and a real number y such that : the value of x square is greater than the value of y and x is smaller than y

(ii) For every real number x, if x is not equal to 0 then there is a real number y such that x times y is equal to 1.

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2(b) Let $I(x)$ be the statement "x has an internet connection" and $C(x, y)$ be the statement "x and y have chatted over the internet," where the domain for the variables x and y consists of all people in the class. Use quantifiers to express each of these statements:

(i) Someone in the class has an internet connection, but has not chatted with anyone in the class.

(ii) There are two people in the class who, between them, have chatted with everyone in the class.

(i) $\exists x (I(x) \wedge \forall y (\neg C(x,y)))$

(ii) $\exists x \exists y (\forall z (C(x,z) \vee C(y,z)))$

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3(a) Suppose the domain of the predicate $P(x, y)$ consists of pairs x and y, where x is 1, 2, or 3, and y is a or b. Write the following propositions using disjunctions and conjunctions (and without quantifiers):

(i) $\exists y\, P(1, y)$

(ii) $\forall x\, \neg P(x, b)$

(i) $P(1,a) \vee P(1,b)$ ✓

(ii) $(\neg P(1,b)) \wedge (\neg P(2,b)) \wedge (\neg P(3,b)) \equiv \neg(P(1,b) \vee P(2,b) \vee P(3,b))$

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3(b) Use quantifiers and predicates with two variables to express the statement: "Some student in this class has visited Alaska, but has not visited any other exotic location." Say clearly what the domains for your variables are.

Domain for x: all students in this class.

Domain for y: all exotic locations.

$P(x,y)$: "x has visited y."

The statement is expressed by:

$\exists x\, (P(x, \text{Alaska}) \wedge \forall y\, (y \neq \text{Alaska} \rightarrow \neg P(x,y)))$ ✓

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4. Use the rules of inference (see next page) to show that the premises $\forall x(P(x) \rightarrow Q(x))$, $\forall x(Q(x) \rightarrow R(x))$, and $\exists x \neg R(x)$ imply the conclusion $\exists x \neg P(x)$. Justify each step.

1	$\forall x\ (P(x) \rightarrow Q(x))$	(Premise)
2	$\forall x\ (Q(x) \rightarrow R(x))$	(Premise)
3	$\exists x\ \neg R(x)$	(Premise)
4	$\neg R(c)$ for some element c	(Existential Instantiation on 3)
5	$Q(c) \rightarrow R(c)$ for the same c	(Universal Instantiation on 2)
6	$\neg Q(c)$ for the same c	(Modus tollens on 4 and 5)
7	$P(c) \rightarrow Q(c)$ for the same c	(Universal Instantiation on 1)
8	$\neg P(c)$ for the same c	(Modus tollens on 6 and 7)
9	$\therefore \exists x\ \neg P(x)$	(Existential Generalization on 8) <conclusion>

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