**1.** Use a truth table to show that $\neg p \to (q \to r)$ and $q \to (p \vee r)$ are logically equivalent. Don't just give the table; say briefly why the table shows this.

| $p$ | $q$ | $r$ | $\neg p$ | $q \to r$ | $\neg p \to (q \to r)$ | $p \vee r$ | $q \to (p \vee r)$ |
|---|---|---|---|---|---|---|---|
| T | T | T | F | T | T | T | T |
| T | T | F | F | F | T | T | T |
| T | F | T | F | T | T | T | T |
| T | F | F | F | T | T | T | T |
| F | T | T | T | T | T | T | T |
| F | T | F | T | F | F | F | F |
| F | F | T | T | T | T | T | T |
| F | F | F | T | T | T | F | T |

These columns agree.

So for any combination of truth values for p, q, r, the two propositions have the same truth value. Thus they are logically equivalent.

**2(a)** Let $S(x)$ be the predicate "$x$ is a student," $F(x)$ the predicate "$x$ is a faculty member," and $A(x, y)$ the predicate "$x$ has asked $y$ a question," where the domain consists of all people associated with OU. Use quantifiers to express these statements:

**(i)** Every student has asked Professor Gross a question.

**(ii)** There is a faculty member who has never been asked a question by a student.

(i) $\forall x\,(S(x) \rightarrow A(x, \text{Professor Gross}))$

(ii) $\exists y\,(F(y) \wedge \forall x\,(S(x) \rightarrow \neg A(x,y)))$

**2(b)** Let $T(x, y)$ mean that $x$ likes cuisine $y$, where the domain for $x$ consists of all students at OU and the domain for $y$ consists of all cuisines. Express each of these statements by a simple English sentence.

**(i)** $\exists x T(x, \text{Korean}) \wedge \forall x T(x, \text{Mexican})$

**(ii)** $\exists x \exists z \forall y (x \neq z \wedge (T(x, y) \leftrightarrow T(z, y)))$

(i) There is an OU student who likes Korean cuisine, and every OU student likes Mexican cuisine.

(ii) There is a pair of students at OU who like exactly the same cuisines as each other.

**3(a)** Show that $\forall x(P(x) \to Q(x))$ and $\forall x P(x) \to \forall x Q(x)$ are not logically equivalent.

Suppose the domain has two elements a, b and $P(a)$ is true, $P(b)$, $Q(a)$, $Q(b)$ are false.

Then $\forall x(P(x) \to Q(x))$ is <u>false</u>, because $P(a) \to Q(a)$ is false.

But $\forall x P(x) \to \forall x Q(x)$ is <u>true</u>, because $\forall x P(x)$ is false.

So in this situation, the two statements have different truth values. Hence, they are not logically equivalent.

**3(b)** Rewrite the following statements so that negations appear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives).

**(i)** $\neg(\exists x \exists y \neg P(x,y) \wedge \forall x \forall y Q(x,y))$

**(ii)** $\neg \exists z \forall y \forall x T(x,y,z)$

(i) $\forall x \forall y\, P(x,y) \vee \exists x \exists y\, \neg Q(x,y)$

(ii) $\forall z \exists y \exists x\, \neg T(x,y,z)$

**4.** Use the rules of inference (see next page) to show that the premises $\forall x(P(x) \rightarrow (Q(x) \wedge S(x)))$ and $\forall x(P(x) \wedge R(x))$ imply the conclusion $\forall x(R(x) \wedge S(x))$. Justify each step.

| | | |
|---|---|---|
| 1 | $\forall x(P(x) \rightarrow (Q(x) \wedge S(x)))$ | hypothesis |
| 2 | $P(c) \rightarrow (Q(c) \wedge S(c))$ | Univ. Instantiation |
| 3 | $\forall x(P(x) \wedge R(x))$ | hypothesis |
| 4 | $P(c) \wedge R(c)$ | Univ. Instantiation |
| 5 | $P(c)$ | Simplification of 4 |
| 6 | $Q(c) \wedge S(c)$ | Modus Ponens on 2, 5 |
| 7 | $S(c)$ | Simplification of 6 |
| 8 | $R(c)$ | Simplification of 4 |
| 9 | $R(c) \wedge S(c)$ | Conjunction of 8, 7 |
| 10 | $\forall x(R(x) \wedge S(x))$ | Univ. Generalization, since c was arbitrary |

**TABLE 1 Rules of Inference.**

| *Rule of Inference* | *Tautology* | *Name* |
|---|---|---|
| $p$<br>$p \to q$<br>$\therefore q$ | $[p \land (p \to q)] \to q$ | Modus ponens |
| $\neg q$<br>$p \to q$<br>$\therefore \neg p$ | $[\neg q \land (p \to q)] \to \neg p$ | Modus tollens |
| $p \to q$<br>$q \to r$<br>$\therefore p \to r$ | $[(p \to q) \land (q \to r)] \to (p \to r)$ | Hypothetical syllogism |
| $p \lor q$<br>$\neg p$<br>$\therefore q$ | $[(p \lor q) \land \neg p] \to q$ | Disjunctive syllogism |
| $p$<br>$\therefore p \lor q$ | $p \to (p \lor q)$ | Addition |
| $p \land q$<br>$\therefore p$ | $(p \land q) \to p$ | Simplification |
| $p$<br>$q$<br>$\therefore p \land q$ | $[(p) \land (q)] \to (p \land q)$ | Conjunction |
| $p \lor q$<br>$\neg p \lor r$<br>$\therefore q \lor r$ | $[(p \lor q) \land (\neg p \lor r)] \to (q \lor r)$ | Resolution |

**TABLE 2 Rules of Inference for Quantified Statements.**

| *Rule of Inference* | *Name* |
|---|---|
| $\forall x P(x)$<br>$\therefore P(c)$ | Universal instantiation |
| $P(c)$ for an arbitrary $c$<br>$\therefore \forall x P(x)$ | Universal generalization |
| $\exists x P(x)$<br>$\therefore P(c)$ for some element $c$ | Existential instantiation |
| $P(c)$ for some element $c$<br>$\therefore \exists x P(x)$ | Existential generalization |