1. (8 points) Find the following partial derivatives.

(a) f_x and f_y where $f(x, y) = y^x$

$$\boxed{\begin{aligned} f_x &= y^x \ln(y) \\ f_y &= x y^{x-1} \end{aligned}}$$

(b) u_{yx} where $u = xy^3e^y$

$$u_y = xy^3e^y + 3xy^2e^y$$

$$\boxed{u_{yx} = y^3e^y + 3y^2e^y}$$

(c) $f_{yyxyyyxy}$ where $f(x, y) = x\tan(y^3)$

$f_{yyxyyyxy} = f_{xxyyyyyy}$ by Clairaut's Theorem

$f_x = \tan(y^3)$

$f_{xx} = 0$ so $\boxed{f_{xxyyyyyy} = 0}$

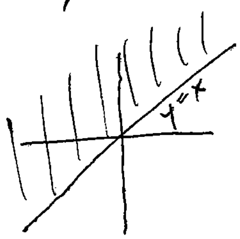
(d) $\dfrac{\partial z}{\partial x}$ where $z = f(xy)$

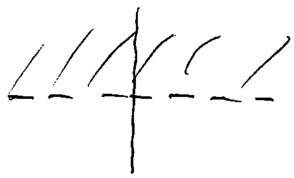
$$\boxed{\frac{\partial z}{\partial x} = f'(xy)\,y}$$

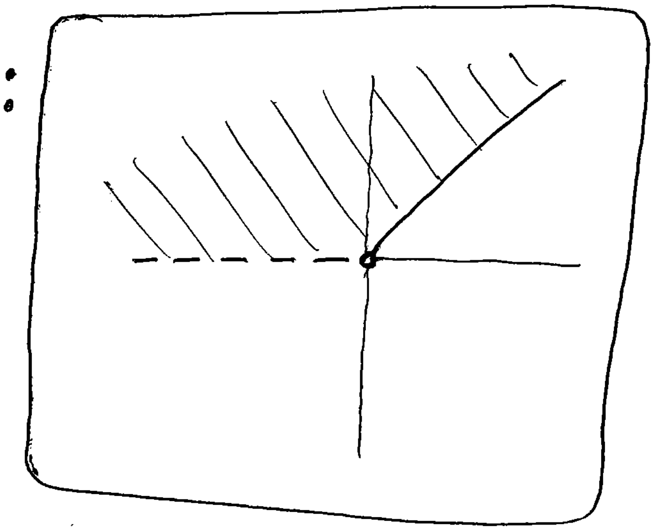
by the one-variable chain rule

2a. (6 points) Sketch the domain of the function $f(x, y) = \ln(y) + 4\sqrt{y-x} - |xy|$. Be sure to indicate which boundary points are in the domain by using dotted/solid lines, and open/closed dots at special points.

$y - x \geq 0$ and $y > 0$:







2b. (2 points) Find the range of $g(x, y) = |x \cos(y)|$.

range = $\boxed{[0, \infty)}$

3. (9 points) A contour map for a function $f(x, y)$ is shown below.

(a) Estimate $f(-3, 3)$ and $f(3, -2)$.

$f(-3,3) \approx 56$
$f(3,-2) \approx 35$

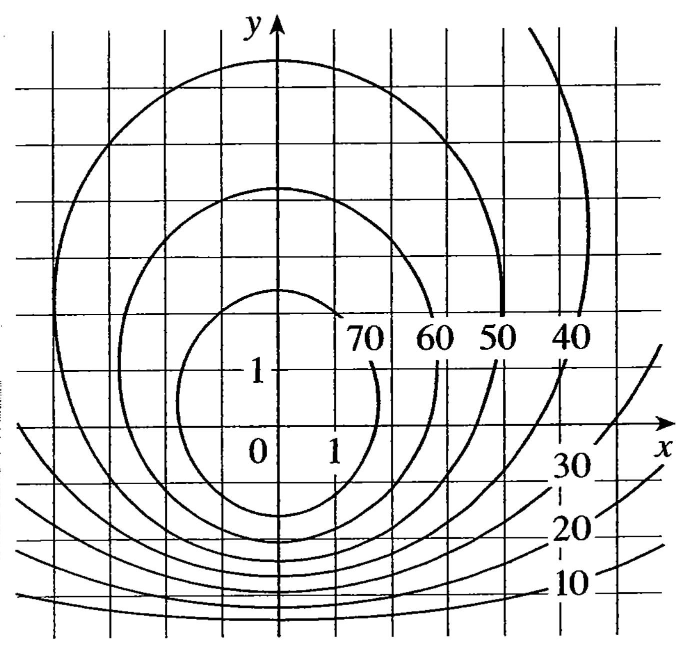
(b) Estimate $f_x(0, -3)$ and $f_y(0, -3)$.

$f_x(0,-3) = 0$
$f_y(0,-3) \approx 30$

(c) Is $f_{xx}(0, -3)$ positive, negative, or zero?

$\boxed{\text{negative}}$

($y = -3$ slice is concave down)



4a. (6 points) Consider the function $f(x,y) = \dfrac{x^4}{x^2+2y^2x^2}$. Write down a continuous function which agrees with $f(x,y)$ away from $(0,0)$. Then find $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^4}{x^2+2y^2x^2}$ and explain why the limit exists.

Away from $(0,0)$, $f(x,y) = \dfrac{x^2}{1+2y^2}$, which is cont.

Hence $\displaystyle\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} \frac{x^2}{1+2y^2}$.

Since $\dfrac{x^2}{1+2y^2}$ is continuous, we can plug in $(0,0)$ to get the limit.

Hence the limit is $\dfrac{0^2}{1+2(0)^2} = 0$.

4b. (6 points) Show that the limit $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2xy^2}{x^2+y^4}$ does not exist. [Hint: try approaching along a parabola.]

Approaching along the x-axis we have

$$\lim_{(x,0)\to(0,0)} \frac{2x(0)^2}{x^2+(0)^4} = \lim_{x\to 0} \frac{0}{x^2} = 0.$$

Approaching along the parabola $x=y^2$ we have

$$\lim_{(y^2,y)\to(0,0)} \frac{2y^2y^2}{(y^2)^2+y^4} = \lim_{(y^2,y)\to(0,0)} 1 = 1.$$

Two different limits $\Rightarrow \displaystyle\lim_{(x,y)\to(0,0)} \frac{2xy^2}{x^2+y^4}$ does not exist.

5a. (6 points) Explain why the function $f(x, y) = e^x \sin(xy)$ is differentiable at $(0, 2)$. Then find either the linearization $L(x, y)$ at this point or the equation of the tangent plane (your choice).

First, $f_x = e^x y\cos(xy) + e^x \sin(xy)$

$f_y = e^x x\cos(xy)$.

These functions are both continuous everywhere, so $f(x,y)$ is differentiable.

$$L(x,y) = \underbrace{f_x(0,2)}_{2}(x-0) + \underbrace{f_y(0,2)}_{0}(y-2) + \underbrace{f(0,2)}_{0}$$

$$\boxed{L(x,y) = 2x}$$

(tangent plane: $z = 2x$)

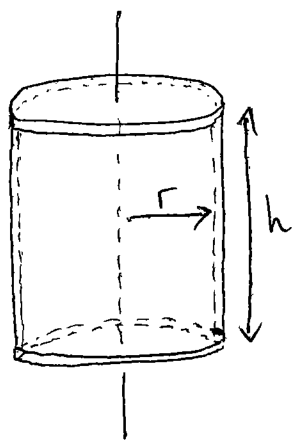
5b. (3 points) Estimate $f(0.01, 1.97)$. [A calculator should not be necessary.]

$$f(0.01, 1.97) \approx L(0.01, 1.97) = \boxed{0.02}$$

6. (8 points) Let $V = \pi r^2 h$. Find dV. Use this to estimate the amount of tin in a closed tin can with diameter 8 cm and height 14 cm if the tin is 0.03 cm thick.

$$dV = \frac{\partial V}{\partial r} dr + \frac{\partial V}{\partial h} dh$$

$$\boxed{dV = 2\pi r h\, dr + \pi r^2 dh}$$



Thickness = 0.03 cm $\Rightarrow$

$dr = 0.03$ cm

$dh = 0.06$ cm

Amount of tin $\approx dV = 2\pi(4)(14)(.03) + \pi(4)^2(.06)$

$$= \boxed{4.32\pi \text{ cm}^3}$$