1(a) Find parametric equations for the line through $(5,1,0)$ that is perpendicular to the plane $2x - y + z = 1$.

(b) In what points does this line intersect the coordinate planes?

(a) direction = normal vector to plane

$= \langle 2, -1, 1 \rangle$

parametric equations:

$$\boxed{\begin{aligned} x(t) &= 5+2t \\ y(t) &= 1-t \\ z(t) &= t \end{aligned}}$$

(b) xy-plane: $z=0$, i.e. $t=0$

so $\boxed{(5,1,0)}$

yz-plane: $x=0$, i.e. $5=-2t$,

$t = -5/2$

so $\boxed{(0, 7/2, -5/2)}$

xz-plane: $y=0$, i.e. $1=t$

so $\boxed{(7,0,1)}$

2. Consider the surface $25y^2 + z^2 = 100 + 4x^2$.

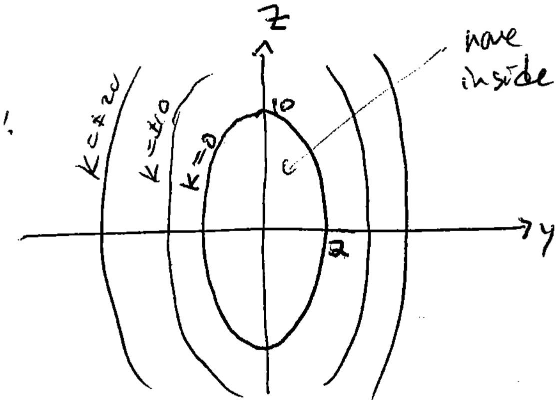
(a) Draw the traces in the planes $x = k$, $y = k$, and $z = k$ for various values of k. Label the curves with their k values, and indicate whether any of them are special.

(b) What kind of surface is this? Carefully draw a sketch.

$x=k$: $25y^2+z^2=100+4k^2$ ellipses!

$k=0$: $\frac{y^2}{4}+\frac{z^2}{100}=1$

others are larger

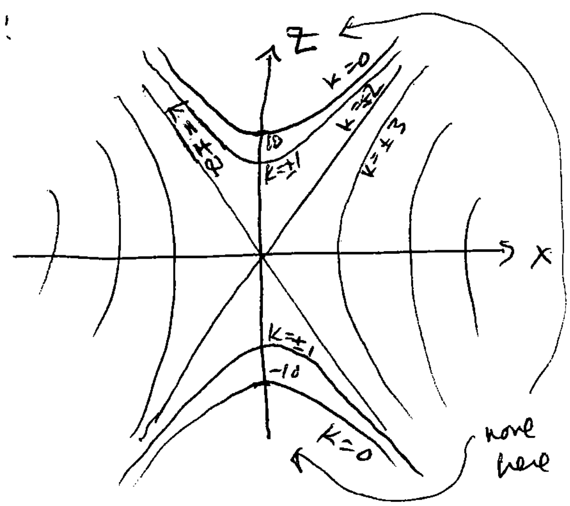


$y=k$: $25k^2-100 = 4x^2-z^2$ hyperbolas!

↑ zero when $k=2$

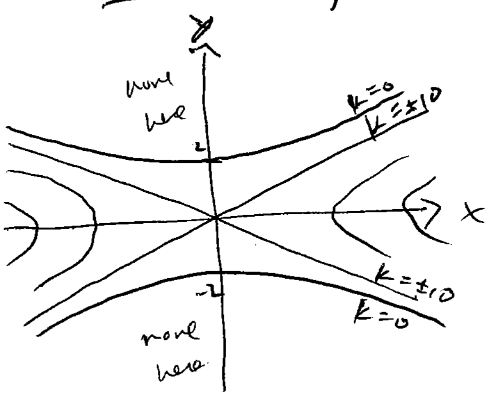
smallest: $k=0$: $-100 = 4x^2-z^2$

no largest value

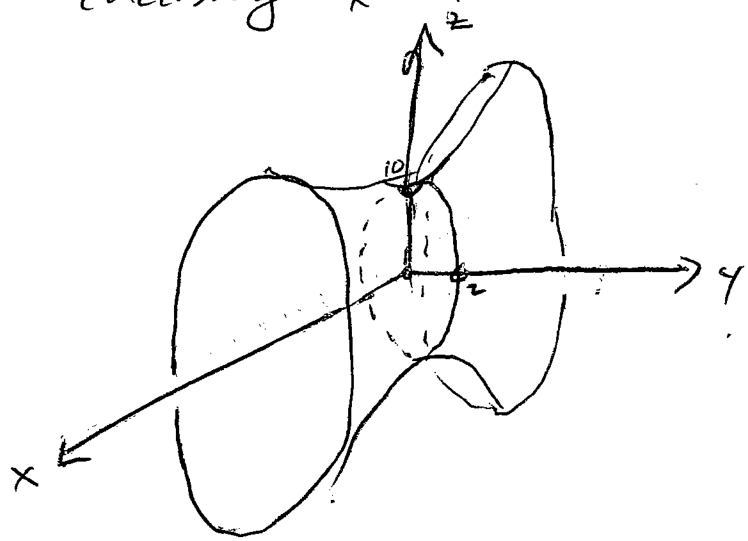


$z=k$: $25y^2-4x^2=100-k^2$ hyperbolas

↑ largest! $k=0$
no smallest value
zero when $k=10$



(b) hyperboloid of one sheet, enclosing x-axis!



3(a) Find an equation of the plane through the points $(3, 1, 2)$, $(2, 4, 6)$, and $(1, -1, 1)$.

(b) Find the cosine of the angle between the planes $2x - 3y = 4z - 1$ and $8y + z + 8x = 12$.

(a) two vectors parallel to plane are:

$\langle 1, -3, -4 \rangle$ and $\langle 2, 2, 1 \rangle$.

normal vector:

$$\langle 1, -3, -4 \rangle \times \langle 2, 2, 1 \rangle = \langle 5, -9, 8 \rangle$$

So $5x - 9y + 8z = C$

plug in $(3, 1, 2)$: $5(3) - 9(1) + 8(2) = C$

$C = 22$

$$\boxed{5x - 9y + 8z = 22}$$

(b) same as angle between normal vectors

$n_1 = \langle 2, -3, -4 \rangle$, $n_2 = \langle 8, 8, 1 \rangle$

$$\cos\theta = \frac{n_1 \cdot n_2}{|n_1||n_2|} = \frac{16 - 24 - 4}{\sqrt{4+9+16}\,\sqrt{64+64+1}}$$

$$= \boxed{\frac{-12}{\sqrt{29}\,\sqrt{129}}}$$

4(a) Find the values of x such that the vectors $\langle 4, x, 6\rangle$ and $\langle 8, x, 2x\rangle$ are orthogonal.

(b) Find the acute angle between two diagonals of a cube. [Draw a picture!]

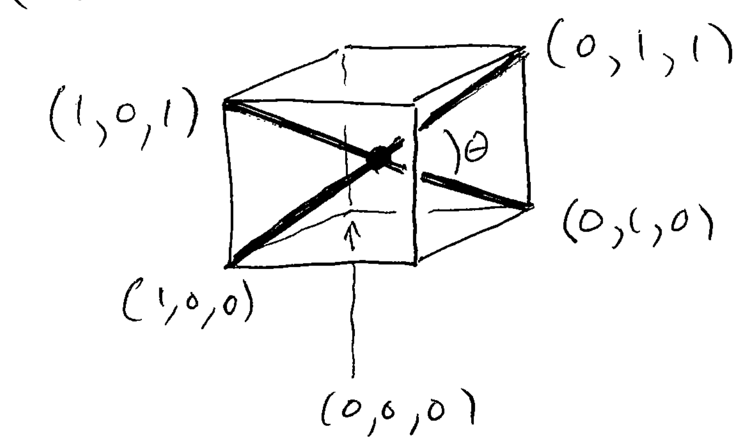
(a) orthogonal $\iff \langle 4, x, 6\rangle \cdot \langle 8, x, 2x\rangle = 0$

$\iff 32 + x^2 + 12x = 0$

$(x+8)(x+4) = 0$

$$\boxed{x = -8 \quad \text{or} \quad x = -4}$$

(b)



diagonals as vectors: $\underset{a}{\langle -1, 1, 1\rangle}, \underset{b}{\langle -1, 1, -1\rangle}$

$\cos\theta = \dfrac{a \cdot b}{|a||b|} = \dfrac{1+1-1}{\sqrt{3}\sqrt{3}} = \dfrac{1}{3}$, angle is acute.

$$\boxed{\theta = \cos^{-1}\left(\tfrac{1}{3}\right)}$$

5(a) Find the volume ofthe parallelepiped determined by the vectors $\mathbf{i}+\mathbf{j}-\mathbf{k}$, $\mathbf{i}-\mathbf{j}+\mathbf{k}$, and $-\mathbf{i}+\mathbf{j}+\mathbf{k}$.

(b) Show that the vectors $\langle 2,3,1\rangle$, $\langle 1,-1,0\rangle$, and $\langle 7,3,2\rangle$ lie within a plane.

(a) vectors $\underbrace{\langle 1,1,-1\rangle}_{a}$, $\underbrace{\langle 1,-1,1\rangle}_{b}$, $\underbrace{\langle -1,1,1\rangle}_{c}$

$$\text{Volume} = |a\cdot(b\times c)|$$
$$= |\langle 1,1,-1\rangle\cdot\langle -2,-2,0\rangle|$$
$$= |-2-2-0| = \boxed{4}$$

(b) this happens $\Leftrightarrow$ the parallelepiped they span has zero volume.

So: $\text{volume} = |\langle 2,3,1\rangle\cdot(\langle 1,-1,0\rangle\times\langle 7,3,2\rangle)|$
$$= |\langle 2,3,1\rangle\cdot\langle -2,-2,10\rangle|$$
$$= |-4-6+10| = 0 \checkmark$$

So they lie within a plane.