Hw # 8 Solutions

The following rules are needed to solve homework from section 6.3 and 6.4.

(**i**) $\log_a x^k = k\log_a x$

(**ii**) $\log_a xy = \log_a x + \log_a y$

(**iii**) $\log_a \dfrac{x}{y} = \log_a x - \log_a y$

(**iv**) $\log_a a^x = x$. In particular, $\ln e^x = x$ for all $x \in \mathbb{R}$ and $e^{\ln x} = x$ for $x > 0$.

(**iv**) $\dfrac{d}{dx}(a^x) = a^x \ln a$, $\dfrac{d}{dx}(\log_a x) = \dfrac{1}{x\ln a}$, $\dfrac{d}{dx}(\ln|x|) = \dfrac{1}{x}$, $\displaystyle\int \frac{1}{x}dx = \ln|x| + C$.

6.3#7 (a) Using rule (iii) first and then rule (ii), we obtain

$$\begin{aligned}\log_2 6 - \log_2 15 + \log_2 20 &= \log_2\left(\frac{6}{15}\right) + \log_2 20\\ &= \log_2\left(\frac{6}{15}.20\right)\\ &= \log_2 8 = 3.\end{aligned}$$

(b) Using rule (iii), we obtain

$$\begin{aligned}\log_3 100 - \log_3 18 - \log_3 50 &= \log_3\left(\frac{100}{18}\right) - \log_3 50\\ &= \log_3\left(\frac{100}{18}.50\right)\\ &= \log_3\left(\frac{1}{9}\right)\\ &= \log_3\left(3^{-2}\right)\\ &= -2\log_3 3 \quad (\text{using rule } (i))\\ &= -2.\end{aligned}$$

6.3#28 (a)

$$\begin{aligned}\ln(x^2 - 1) &= 3 \Longrightarrow x^2 - 1 = e^3\\ &\Longrightarrow x^2 = e^3 + 1\\ &\Longrightarrow x = \pm\sqrt{e^3 + 1}.\end{aligned}$$

(b) Put $y = e^x$, then $e^{2x} - 3e^x + 2 = 0$ reduces to

$$\begin{aligned}y^2 - 3y + 2 &= 0\\ (y-1)(y-2) &= 0\\ (e^x - 1)(e^x - 2) &= 0.\end{aligned}$$

$e^x - 1 = 0$ gives $e^x = 1$ and hence $x = \ln 1 = 0$. Next, $e^x - 2 = 0$ gives $e^x = 2$ and hence $x = \ln 2$. Therefore, $x = 0$ or $\ln 2$.

6.4#8 Using Chain Rule, we have

$$\begin{aligned} f'(x) &= \frac{1}{xe^x \ln 5} \frac{d}{dx}(xe^x) \\ &= \frac{1}{xe^x \ln 5}(e^x + xe^x) \\ &= \frac{e^x(1+x)}{xe^x \ln 5} = \frac{1+x}{x \ln 5}. \end{aligned}$$

6.4#52 Taking ln on both sides, we obtain

$$\begin{aligned} \ln y &= \ln(\sin x)^{\ln x} \\ \ln y &= \ln x \ln \sin x \quad \text{(using rule } (i)) \end{aligned}$$

Differentiating both sides with respect to x (using Chain Rule and product rule), we get

$$\frac{1}{y} y' = \ln x . \frac{1}{\sin x} . \cos x + \ln \sin x . \frac{1}{x}$$

Solving for y', we get

$$\begin{aligned} y' &= y \left(\ln x \cot x + \frac{\ln \sin x}{x} \right) \\ &= (\sin x)^{\ln x} \left(\ln x \cot x + \frac{\ln \sin x}{x} \right). \end{aligned}$$

6.4#72 Put $u = 5x + 1$. Then $du = 5dx$. New limits are $u = 1$ and $u = 16$. Thus

$$\begin{aligned} \int_0^3 \frac{dx}{5x+1} &= \int_1^{16} \frac{1}{u} \frac{du}{5} = \frac{1}{5} \int_1^{16} \frac{du}{u} \\ &= \frac{1}{5} \ln |u| \,\Big|_1^{16} = \frac{1}{5}[\ln 16 - \ln 1] \\ &= \frac{1}{5} \ln 16. \end{aligned}$$