§3.9

Find the most general antiderivative of the function. (Check your answer by differentiation.)

6. $f(x) = x(2-x)^2$
$= x(4-4x+x^2)$
$= x^3 - 4x^2 + 4x$

Recall: $\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = x^n \Rightarrow \frac{x^{n+1}}{n+1}$ is an antiderivative of x^n.

Consider an antiderivative:

$F(x) = \left(\frac{x^4}{4}\right) - 4\left(\frac{x^3}{3}\right) + 4\left(\frac{x^2}{2}\right) = \frac{1}{4}x^4 - \frac{4}{3}x^3 + 2x^2$

[Check: $F'(x) = \frac{1}{4}(4x^3) - \frac{4}{3}(3x^2) + 2(2x) = x^3 - 4x^2 + 4x = f(x)$ ✓]

As antiderivatives are determined upto a constant:

$\boxed{F(x) = \frac{1}{4}x^4 - \frac{4}{3}x^3 + 2x^2 + C}$ is the most general antiderivative, where C is some constant.

10. $f(x) = \pi^2$ [Think of π^2 as $\pi^2 x^0$] Consider an antiderivative:

$F(x) = \pi^2\left(\frac{x^1}{1}\right) = \pi^2 x$

[Check: $F'(x) = \pi^2 = f(x)$ ✓]

As antiderivatives are determined upto a constant:

$\boxed{F(x) = \pi^2 x + C}$ is the most general antiderivative, where C is some constant.

18. $f(x) = 2\sqrt{x} + 6\cos x = 2x^{\frac{1}{2}} + 6\cos x$

Recall: $\frac{d}{dx}(\sin x) = \cos x \Rightarrow \sin x$ is an antiderivative of $\cos x$.

Consider an antiderivative:

$F(x) = 2\left(\frac{x^{3/2}}{3/2}\right) + 6(\sin x) = 2 \cdot \frac{2}{3}x^{3/2} + 6\sin x = \frac{4}{3}x^{3/2} + 6\sin x$

[Check: $F'(x) = \frac{4}{3}\left(\frac{3}{2}x^{\frac{1}{2}}\right) + 6(\cos x) = 2x^{\frac{1}{2}} + 6\cos x = f(x)$ ✓]

As antiderivatives are determined upto a constant:

$\boxed{F(x) = \frac{4}{3}x^{3/2} + 6\sin x + C}$ is the most general antiderivative, where C is some constant.

Find f

28. $f'(x) = 5x^4 - 3x^2 + 4$, $f(-1) = 2$

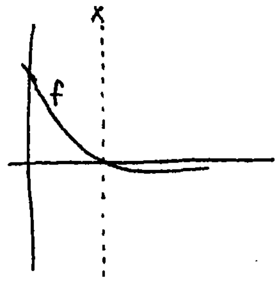
$f(x) = 5\left(\frac{x^5}{5}\right) - 3\left(\frac{x^3}{3}\right) + 4\left(\frac{x^1}{1}\right) + C = x^5 - x^3 + 4x + C$, where C is some constant.

$f(-1) = 2 \Rightarrow$
$2 = (-1)^5 - (-1)^3 + 4(-1) + C$
$2 = -1 - (-1) - 4 + C$
$6 = C$

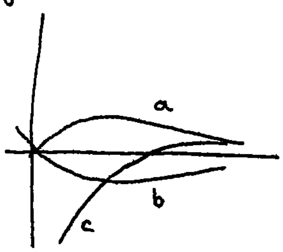
Therefore, $\boxed{f(x) = x^5 - x^3 + 4x + 6}$

The graph of a function f is shown. Which graph is the antiderivative of f and why?

44.



Ans: f is the derivative of a function with positive derivateve before x and negative derivative after x. So, f is the derivative of a function that increases before x and decreases after x [Remember to consider increasing / decreasing as you read the function from left to right.]
The only function that fits this criteria is $\boxed{a}$.



[b decreases, then increases.
c increases only]

54. A particle is moving with the given data. Find the position of the particle.

54. $a(t) = 3\cos t - 2\sin t$ $\quad s(0) = 0 \quad v(0) = 4$

Recall: $\begin{cases} s'(t) = v(t) \\ v'(t) = a(t) \end{cases}$ This may be found in the section "Rectilinear motion" on page 272.

$a(t) = v'(t) = 3\cos t - 2\sin t$

$v(t) = 3\sin t - 2(-\cos t) + C = 3\sin t + 2\cos t + C$, where C is some constant.

$v(0) = 4 \Rightarrow 4 = 3\sin(0) + 2\cos(0) + C$

$4 = 2 + C$

$2 = C$

$v(t) = 3\sin t + 2\cos t + 2$

[check: $v'(t) = 3(\cos t) + 2(-\sin t) = 3\cos t - 2\sin t = a(t)$ ✓]

$v(t) = s'(t) = 3\sin t + 2\cos t + 2$

$s(t) = 3(-\cos t) + 2(\sin t) + 2\left(\frac{t^1}{1}\right) + C = -3\cos t + 2\sin t + 2t + C$, where C is some constant.

$s(0) = 0 \Rightarrow 0 = -3\cos(0) + 2\sin(0) + 2(0) + C$

$0 = -3 + C$

$3 = C$

So $\boxed{s(t) = -3\cos t + 2\sin t + 2t + 3}$

[Check: $s'(t) = -3(-\sin t) + 2(\cos t) + 2$
$= 3\sin t + 2\cos t + 2$ ✓]