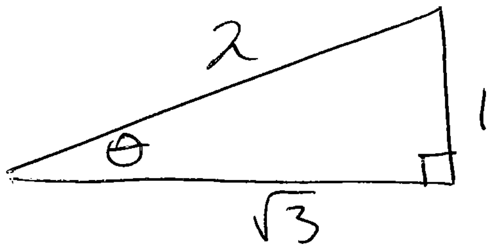
1(a) Evaluate:

(i) $\tan(\cos^{-1}(\frac{\sqrt{3}}{2}))$

$$\cos\theta = \frac{\sqrt{3}}{2}$$



$$\boxed{\tan\theta = \frac{1}{\sqrt{3}}}$$

(ii) $\sinh(\ln 3)$

$$= \frac{1}{2}\left(e^{\ln 3} - e^{-\ln 3}\right)$$

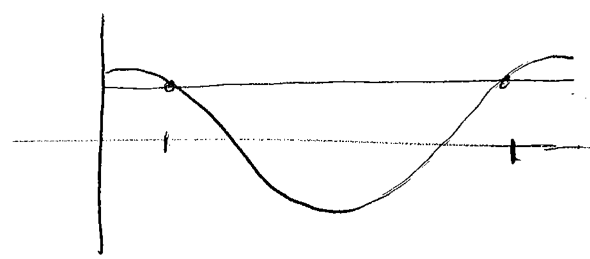
$$= \frac{1}{2}\left(3 - e^{\ln(3^{-1})}\right)$$

$$= \boxed{\frac{1}{2}\left(3 - \frac{1}{3}\right)}$$

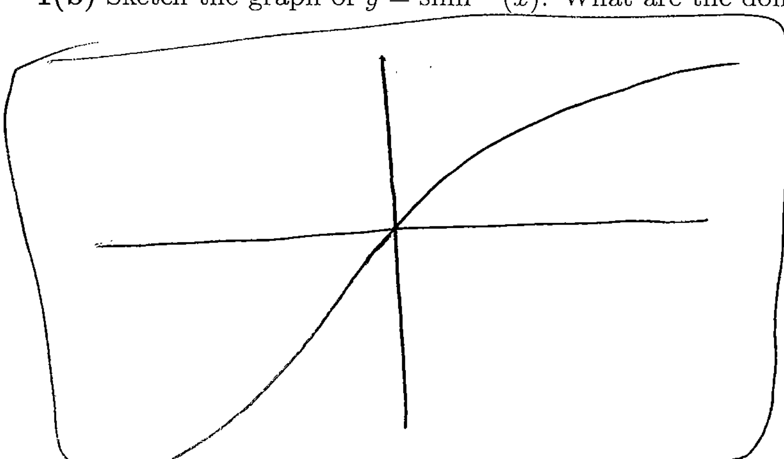
(iii) $\cos^{-1}(\cos(\frac{7\pi}{4}))$

$$= \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$$

$$= \boxed{\frac{\pi}{4}}$$



1(b) Sketch the graph of $y = \sinh^{-1}(x)$. What are the domain and range of this function?



domain and range are both $\mathbb{R}$

(all real numbers)

2. Find the limits, and give your reasoning:

(a) $\lim_{x\to 0^+} x\ln x$ $= 0\cdot(-\infty)$ indeterminate

$= \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}}$ indet type $\frac{-\infty}{\infty}$

$\overset{L'H}{=} \lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-1}{x^2}} = \lim_{x\to 0^+} -x = \boxed{0}$

(b) $\lim_{x\to 0} \frac{e^x - 1 - x}{2x^2}$ indeterminate, type $\frac{0}{0}$

$\overset{L'H}{=} \lim_{x\to 0} \frac{e^x - 1}{4x}$ indeterminate, type $\frac{0}{0}$

$\overset{L'H}{=} \lim_{x\to 0} \frac{e^x}{4} = \boxed{\frac{1}{4}}$

(c) $\lim_{x\to\pi/2} \frac{1-\sin x}{\csc x}$ $\frac{1-1}{1} = \boxed{0}$

3. Evaluate the integrals:

(a) $\int_{\pi/2}^{\pi} \sin^5 x \cos^4 x \, dx$

$$= \int_{\pi/2}^{\pi} (1-\cos^2 x)^2 \cos^4 x \sin x \, dx$$

$$u = \cos x, \quad du = -\sin x \, dx$$

$$= -\int_0^{-1} (1-u^2)^2 u^4 \, du$$

$$= -\int_0^{-1} (u^4 - 2u^6 + u^8) \, du$$

$$= -\left[\frac{1}{5}u^5 - \frac{2}{7}u^7 + \frac{1}{9}u^9 \right]_0^{-1}$$

$$= \boxed{\frac{1}{5} - \frac{2}{7} + \frac{1}{9}}$$

(b) $\int_0^1 t \cosh t \, dt$

Parts: $u = t$ $\quad v = \sinh t$

$du = dt$ $\quad dv = \cosh t \, dt$

$$t \sinh t \Big|_0^1 - \int_0^1 \sinh t \, dt$$

$$\sinh(1) - \left[\cosh t \right]_0^1$$

$$\boxed{\sinh(1) - \cosh(1) + \cosh(0)}$$

$$\left(= 1 - \frac{1}{e} \right)$$

4(a) Find $f'(x)$ if $f(x) = \tanh(1 + e^{2x})$

$$\boxed{f'(x) = \operatorname{sech}^2(1+e^{2x}) \cdot e^{2x} \cdot 2}$$

(b) Find $\int \sinh x \cosh^2 x \, dx$

$u = \cosh x, \quad du = \sinh x \, dx$

$$\int u^2 du = \frac{1}{3}u^3 + C$$

$$\boxed{\frac{1}{3}\cosh^3 x + C}$$

(c) Evaluate $\int_0^{\pi} \sin^4(3t)\, dt$ $= \int_0^{\pi} \left(\frac{1}{2}(1-\cos(6t))\right)^2 dt$

$$= \int_0^{\pi} \frac{1}{4}\left(1 - 2\cos(6t) + \cos^2(6t)\right) dt$$

$$= \int_0^{\pi} \frac{1}{4}\left(1 - 2\cos(6t)\right) dt + \frac{1}{4}\int_0^{\pi} \frac{1}{2}(1 + \cos(12t))\, dt$$

$$= \frac{1}{4}\left[t - \frac{2}{6}\sin(6t)\right]_0^{\pi} + \frac{1}{8}\left[t + \frac{1}{12}\sin(12t)\right]_0^{\pi}$$

$$= \frac{\pi}{4} + \frac{\pi}{8} = \boxed{\frac{3\pi}{8}}$$

$\sin^2 x + \cos^2 x = 1$
$1 + \tan^2 x = \sec^2 x$
$\sin^2 x = \frac{1}{2}(1 - \cos(2x))$
$\cos^2 x = \frac{1}{2}(1 + \cos(2x))$
$\sin(2x) = 2 \sin x \cos x$
$\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$
$\sin A \cos B = \frac{1}{2}(\sin(A - B) + \sin(A + B))$
$\sin A \sin B = \frac{1}{2}(\cos(A - B) - \cos(A + B))$
$\cos A \cos B = \frac{1}{2}(\cos(A - B) + \cos(A + B))$

Extra Credit Evaluate $\int_0^{1/2} \cos^{-1} x \, dx$. Parts: $u = \cos^{-1} x$ $\quad v = x$

$du = \frac{-1}{\sqrt{1-x^2}} dx \quad dv = dx$

$$x \cos^{-1} x \Big|_0^{\frac{1}{2}} - \int_0^{\frac{1}{2}} \frac{-x}{\sqrt{1-x^2}} dx$$

Sub: $u = 1 - x^2$

$du = -2x \, dx$

$$x \cos^{-1} x \Big|_0^{\frac{1}{2}} - \frac{1}{2} \int_1^{3/4} \frac{1}{\sqrt{u}} du$$

$$\frac{1}{2} \cos^{-1}\left(\frac{1}{2}\right) - \frac{1}{2}\left[2u^{\frac{1}{2}}\right]_1^{3/4}$$

$$\frac{\pi}{6} - (3/4)^{1/2} + 1$$

$$\boxed{\frac{\pi}{6} - \frac{\sqrt{3}}{2} + 1}$$