1(a) Find the inverse function $f^{-1}(x)$ when $f(x) = \log_{10}(1 + \frac{1}{x})$.

$$y = \log_{10}(1 + \tfrac{1}{x})$$

$$10^y = 1 + \frac{1}{x}$$

$$\frac{1}{x} = 10^y - 1$$

$$x = \frac{1}{10^y - 1}$$

$$\boxed{f^{-1}(x) = \frac{1}{10^x - 1}}$$

(b) Find the domain of $f(x) = \ln(x^2 - 2x)$.

when $x^2 - 2x > 0$.

$$x^2 - 2x = 0$$
$$x(x-2) = 0$$
$$x = 0, 2$$

$+ \quad - \quad +$

$0 \quad 2$

So $x^2 - 2x > 0$ when

$x < 0$ or $x > 2$

$\boxed{\text{domain: } (-\infty, 0) \cup (2, \infty)}$

(c) Evaluate $\displaystyle\int_1^{e^3} \frac{(\ln x)^2}{x}\, dx$.

$u = \ln x$, $du = \frac{1}{x} dx$

$$= \int_0^3 u^2 du = \frac{1}{3}u^3 \Big|_0^3$$

$$= \frac{1}{3}(3)^3 - 0$$

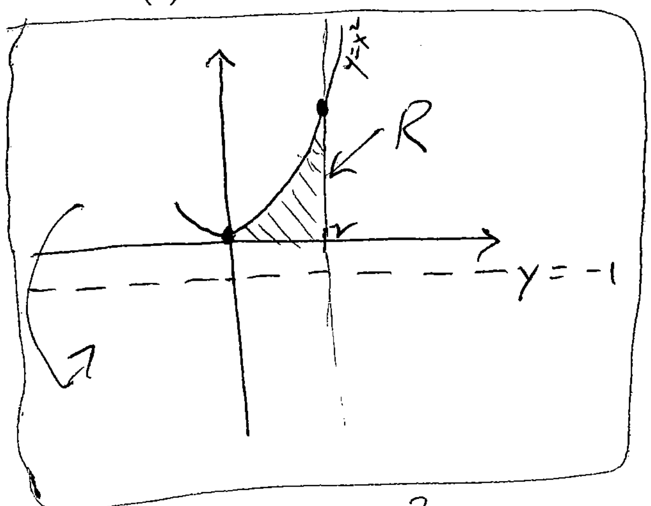
$$= \boxed{9}$$

2. Find the volume of the solid generated when the region R bounded by $y = x^2$, $y = 0$, and $x = 2$ is rotated about the line $y = -1$, as follows.

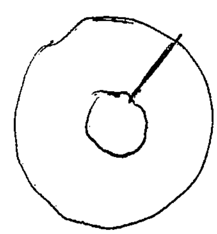
(a) In the xy-plane, draw carefully the region R and the axis of rotation.

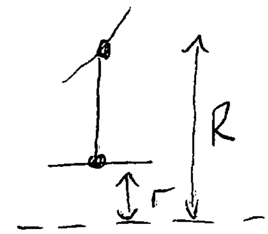
(b) Decide whether to use washers or shells, draw the typical washer or shell for this example, and find its area.

(c) Find the volume of the solid.



washers!





$R = x^2+1, \quad r = 1$

$$\boxed{A(x) = \pi(x^2+1)^2 - \pi(1)^2}$$

$$\text{Volume} = \int_0^2 \left(\pi(x^2+1)^2 - \pi\right) dx = \int_0^2 \pi\left(x^4+2x^2+\cancel{1}-\cancel{1}\right) dx$$

$$= \pi\left[\frac{x^5}{5} + \frac{2x^3}{3}\right]_0^2 = \pi\left(\frac{32}{5}+\frac{16}{3}\right) = \boxed{\frac{176}{15}\pi}$$

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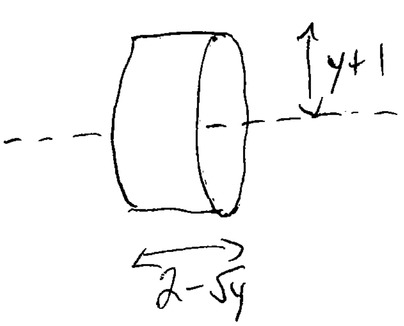
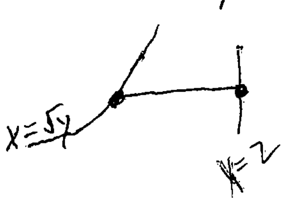
shells!

$$\boxed{A(y) = 2\pi(y+1)(2-\sqrt{y})}$$



$$\text{Volume} = \int_0^4 2\pi\left(2y+2-y^{3/2}-y^{1/2}\right) dy$$

$$= 2\pi\left[y^2+2y-\frac{2}{5}y^{5/2}-\frac{2}{3}y^{3/2}\right]_0^4$$



$$= 2\pi\left[16+8-\frac{2}{5}(4)^{5/2}-\frac{2}{3}(4)^{3/2}\right]$$

$$= 2\pi\left[24-\frac{64}{5}-\frac{16}{3}\right] = \boxed{\frac{176}{15}\pi}$$

3(a) Find the average value of $f(x) = \sqrt{x}$ on the interval $[1, 4]$.

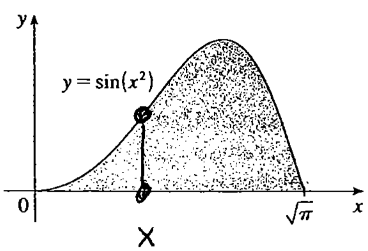
$$f_{ave} = \frac{1}{4-1} \int_1^4 \sqrt{x}\, dx$$

$$= \frac{1}{3} \int_1^4 x^{1/2}\, dx = \frac{1}{3} \cdot \frac{2}{3} x^{3/2} \Big|_1^4$$

$$= \frac{2}{9}(4)^{3/2} - \frac{2}{9}(1) = \frac{2}{9}(8) - \frac{2}{9}$$

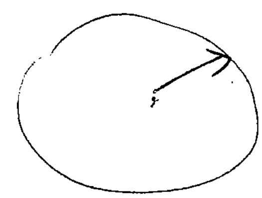
$$= \boxed{\frac{14}{9}}$$

(b) Let S be the solid obtained by rotating the region shown about the x-axis. Set up, but do not evaluate, an integral representing the volume of S using disks. Why are disks preferable to shells in this case?



$\updownarrow \sin(x^2)$

Disk:

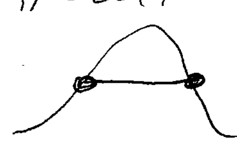


$r = \sin(x^2)$

$$A(x) = \pi(\sin(x^2))^2$$

$$\boxed{\text{Volume} = \int_0^{\sqrt{\pi}} \pi(\sin(x^2))^2\, dx}$$

Shells are more difficult because the endpoints



are hard to determine - must solve for x in $y = \sin(x^2)$.

4(a) Use logarithmic differentiation to find $\frac{dy}{dx}$ where $y = \sqrt{x}e^{x^2}(x^2+1)^{10}$.

$$y = \sqrt{x}\, e^{x^2} (x^2+1)^{10}$$

$$\ln y = \ln(\sqrt{x}) + \ln(e^{x^2}) + \ln((x^2+1)^{10})$$

$$\ln y = \frac{1}{2}\ln x + x^2 + 10 \ln(x^2+1)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2x} + 2x + \frac{20x}{x^2+1}$$

$$\frac{dy}{dx} = y\left(\frac{1}{2x} + 2x + \frac{20x}{x^2+1}\right)$$

$$\boxed{\frac{dy}{dx} = \sqrt{x}\, e^{x^2}(x^2+1)^{10}\left(\frac{1}{2x} + 2x + \frac{20x}{x^2+1}\right)}$$

(b) Find the derivative of $F(t) = e^{t\sin(2t)}$.

$$\boxed{F'(t) = e^{t\sin(2t)} \cdot (2t\cos(2t) + \sin(2t))}$$

Extra Credit Find the derivative of $y = e^{e^{e^{e^x}}}$.

$$\boxed{\frac{dy}{dx} = e^{e^{e^{e^x}}} \cdot e^{e^{e^x}} \cdot e^{e^x} \cdot e^x}$$

chain rule three times