1(a) If $r'(t)$ is the rate of change of the radius of a spherical balloon measured in centimeters per second, what does the integral $\int_1^2 r'(t)\,dt$ represent, and what are its units?

It represents the increase in radius between the first second and the second second. Units are centimeters.

(b) Find the derivatives of $g(x) = \int_1^x (1-v^2)^{10}\,dv$ and $h(x) = \int_1^{\cos x} (1-v^2)^{10}\,dv$.

$g'(x) = (1-x^2)^{10}$

$h(x) = g(\cos x)$, $h'(x) = g'(\cos x)\frac{d}{dx}\cos x$

$\boxed{h'(x) = (1-\cos^2 x)^{10}(-\sin x)}$

$= (\sin^2 x)^{10}(-\sin x)$

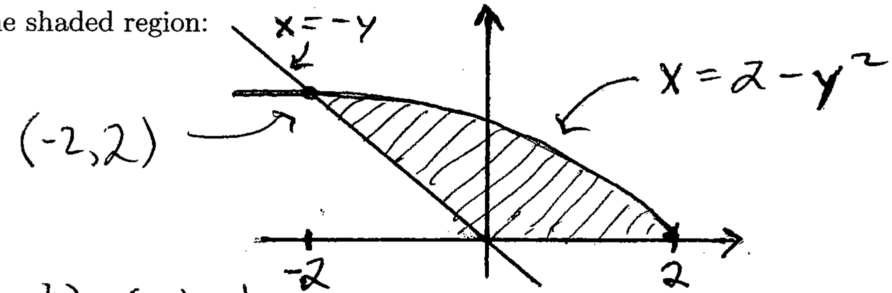
$= -\sin^{21} x$

(c) Write down two different antiderivatives of $f(x) = \cos(\sqrt{x-1})$.

$\int_0^x \cos(\sqrt{t-1})\,dt$

$\int_1^x \cos(\sqrt{t-1})\,dt$

2(a) Find the area of the shaded region:



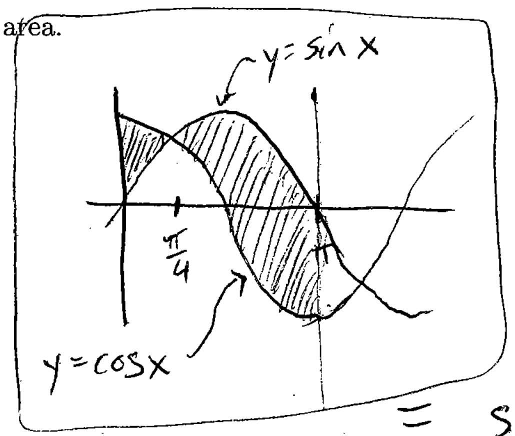
$$\text{Area} = \int_0^2 (2-y^2)-(-y)\,dy$$

$$= \int_0^2 (2-y^2+y)\,dy = 2y - \tfrac{1}{3}y^3 + \tfrac{1}{2}y^2 \Big|_0^2$$

$$= 2(2) - \tfrac{1}{3}(2)^3 + \tfrac{1}{2}(2)^2$$

$$= 4 - \tfrac{8}{3} + 2 = \boxed{10/3}$$

(b) Sketch carefully the region or regions whose area is given by $\int_0^\pi |\cos x - \sin x|\,dx$. Then find this area.



$$\text{Area} = \int_0^{\pi/4} (\cos x - \sin x)\,dx + \int_{\pi/4}^{\pi} (\sin x - \cos x)\,dx$$

$$= \sin x + \cos x \Big|_0^{\pi/4} + -\cos x - \sin x \Big|_{\pi/4}^{\pi}$$

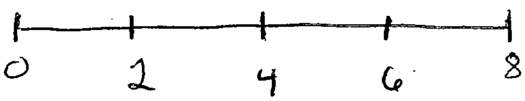
$$= \sin(\tfrac{\pi}{4}) + \cos(\tfrac{\pi}{4}) - \sin(0) - \cos(0) \quad - \cos(\pi) - \sin(\pi) + \cos(\tfrac{\pi}{4}) + \sin(\tfrac{\pi}{4})$$

$$= \boxed{2\sqrt{2}}$$

3. Use four rectangles to estimate the area under the curve $y = \sqrt{800 - x^3}$ between $x = 0$ and $x = 8$ as follows.

(a) Write down the sample points (choose left or right, and say ~~which~~).

$$\boxed{\text{Right}: \quad x_1^* = 2, \quad x_2^* = 4, \quad x_3^* = 6, \quad x_4^* = 8}$$



(b) Write out a complete numerical expression for the area estimate. You do not need to evaluate the number.

$$\Delta x = \frac{8}{4} = 2$$

$$\boxed{\text{Area} = 2\sqrt{800 - (2)^3} + 2\sqrt{800 - (4)^3} + 2\sqrt{800 - (6)^3} + 2\sqrt{800 - (8)^3}}$$

(c) Express part (b) using sigma notation.

$$x_i^* = 2i$$

$$\boxed{\text{Area} = \sum_{i=1}^{4} 2\sqrt{800 - (2i)^3}}$$

(d) Write down an expression for the area estimate given by using n rectangles (using sigma notation or not, as you like).

$$\Delta x = \frac{8}{n}, \quad x_i^* = \frac{8i}{n}$$

$$\boxed{\sum_{i=1}^{n} \frac{8}{n}\sqrt{800 - \left(\frac{8i}{n}\right)^3}}$$

4. Evaluate:

(a) $\displaystyle\int_{\pi^2}^{4\pi^2} \frac{1}{\sqrt{x}} \sin(\sqrt{x})\, dx$

$u = \sqrt{x} = x^{1/2}$

$du = \frac{1}{2}x^{-1/2}dx = \frac{1}{2\sqrt{x}}\,dx$

$\displaystyle\int_{\pi}^{2\pi} 2\sin(u)\,du = -2\cos u \Big|_{\pi}^{2\pi}$

$= -2\cos(2\pi) + 2\cos(\pi)$

$= -2 - 2 = \boxed{-4}$

(b) $\displaystyle\int (4-3x)^8\, dx$

$u = 4 - 3x$

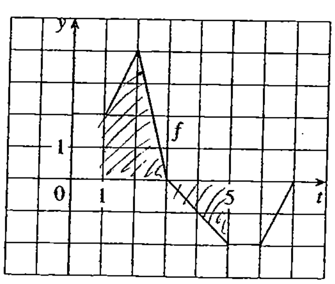
$du = -3dx$

$dx = \frac{-1}{3}du$

$\displaystyle\int \frac{-1}{3} u^8 du = \frac{-1}{3}\cdot\frac{1}{9}u^9 + C$

$= \boxed{\frac{-1}{27}(4-3x)^9 + C}$

(c) $g(5)$ where $g(x) = \displaystyle\int_1^x f(t)\,dt$ and f is as shown:



$g(5) = 5 - 2$

$= \boxed{3}$