

HW 14

8.4 # 4, 8, 10, 22, 28

$$\#4. (a) \frac{x^3}{x^2+4x+3} = x-4 + \frac{13x+12}{x^2+4x+3} = x-4 + \frac{13x+12}{(x+3)(x+1)}$$

$$= x-4 + \frac{A}{x+3} + \frac{B}{x+1}$$

$$(b) \frac{2x+1}{(x+1)^3(x^2+4)^2} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3} + \frac{Dx+E}{x^2+4} + \frac{Fx+G}{(x^2+4)^2}$$

$$\#8 \int \frac{r^2}{r+4} dr = \int (r-4 + \frac{16}{r+4}) dr$$

$$= \frac{1}{2}r^2 - 4r + 16 \ln|r+4| + C$$

$$\#10 \int \frac{1}{(t+4)(t-1)} dt = \int \left(\frac{A}{t+4} + \frac{B}{t-1} \right) dt$$

$$\left. \begin{aligned} A(t-1) + B(t+4) &= 1 \\ A &= -\frac{1}{5} \quad B = \frac{1}{5} \end{aligned} \right\} = -\frac{1}{5} \ln|t+4| + \frac{1}{5} \ln|t-1| + C$$

$$= \frac{1}{5} \ln \left| \frac{t-1}{t+4} \right| + C$$

$$\#22. \int \frac{ds}{s^2(s-1)^2} = \int \left(\frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-1} + \frac{D}{(s-1)^2} \right) ds$$

$$= A \ln|s| + \frac{-B}{s} + C \ln|s-1| + \frac{-D}{s-1} + E$$

$$As(s-1)^2 + B(s-1)^2 + Cs^2(s-1) + Ds^2 = 1$$

$$B=1, D=1$$

$$s=2: 2A + 1 + 4C + 4 = 1 \quad A + 2C = -2 \Rightarrow A = -C \Rightarrow A=2$$

$$s=-1: -4A + 4 - 2C + 1 = 1 \quad 2A + C = 2 \Rightarrow C = -2$$

$$\boxed{2 \ln|s| - \frac{1}{s} - 2 \ln|s-1| - \frac{1}{s-1} + E}$$

$$\#28 \quad \int \frac{x^2 - 2x - 1}{(x-1)^2(x^2+1)} dx = \int \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+1} dx$$

$$A(x-1)(x^2+1) + B(x^2+1) + (Cx+D)(x-1)^2 = x^2 - 2x - 1$$

$$x=1: \quad B = -1$$

$$x=0: \quad -A - 1 + D = -1 \Rightarrow D = A$$

$$x=2: \quad 5A + (-5) + 2C + A = -1 \Rightarrow 3A + C = 2$$

$$x=-1: \quad -4A - 2 - 4C + 4A = 2 \Rightarrow C = -1$$

$$\Rightarrow A = 1$$

$$\int \left(\frac{1}{x-1} + \frac{-1}{(x-1)^2} + \frac{-x}{x^2+1} + \frac{1}{x^2+1} \right) dx$$

$$= \ln|x-1| + \frac{1}{x-1} - \frac{1}{2} \ln|x^2+1| + \tan^{-1}(x) + C$$