HW #13

8.2 #2, 20, 34
8.3 #6, 14

8.2 #2. $\int \sin^6 x \cos^3 x\,dx = \int \sin^6 x(1-\sin^2 x)\cos x\,dx$

$u = \sin x$
$du = \cos x\,dx$

$= \int u^6(1-u^2)\,du$

$= \int u^6 - u^8\,du = \frac{1}{7}u^7 - \frac{1}{9}u^9 + C$

$= \frac{1}{7}\sin^7 x - \frac{1}{9}\sin^9 x + C$

20. $\int \cos^2 x \sin 2x\,dx = \int \frac{1}{2}(1+\cos 2x)\sin 2x\,dx$

$u = \cos 2x$
$du = -2\sin 2x\,dx$

$= -\frac{1}{4}\int (1+u)\,du = -\frac{1}{4}\left(u + \frac{1}{2}u^2\right) + C$

$= -\frac{1}{4}\left(\cos 2x + \frac{1}{2}\cos^2 2x\right) + C$

34. $\int \tan^2 x \sec x\,dx$

$\begin{pmatrix} u = \tan x & dv = \sec x \tan x\,dx \\ du = \sec^2 x\,dx & v = \sec x \end{pmatrix}$

$= \tan x \sec x - \underbrace{\int \sec^3 x\,dx}_{\text{see Ex. 8 in book}} = \tan x \sec x - \frac{1}{2}(\sec x \tan x + \ln|\sec x + \tan x|) + C$

8.3 #6. $\int_1^2 \frac{\sqrt{x^2-1}}{x}\,dx \qquad x = \sec\theta \qquad dx = \sec\theta\tan\theta\,d\theta$

$= \int_0^{\pi/3} \frac{\sqrt{\sec^2\theta - 1}}{\sec\theta}\cdot\sec\theta\tan\theta\,d\theta$

$= \int_0^{\pi/3} \sqrt{\tan^2\theta}\cdot\tan\theta\,d\theta = \int_0^{\pi/3} \tan^2\theta\,d\theta$

$= \int_0^{\pi/3}(\sec^2\theta - 1)\,d\theta = \tan\theta - \theta\Big|_0^{\pi/3} = \boxed{\sqrt{3} - \frac{\pi}{3}}$

8.3 #14 $\displaystyle\int \frac{du}{u\sqrt{5-u^2}}$

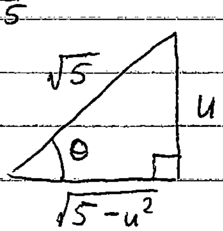
$$\left(u=\sqrt{5}\sin\theta \qquad du=\sqrt{5}\cos\theta\, d\theta\right)$$

$$\int \frac{du}{u\sqrt{5-u^2}} = \int \frac{\sqrt{5}\cos\theta\, d\theta}{\sqrt{5}\sin\theta\sqrt{5-5\sin^2\theta}} = \int \frac{\cos\theta\, d\theta}{\sin\theta\cdot\sqrt{5\cos^2\theta}}$$

$$= \int \frac{\cos\theta\, d\theta}{\sqrt{5}\sin\theta\cos\theta} = \frac{1}{\sqrt{5}}\int \frac{d\theta}{\sin\theta} = \frac{1}{\sqrt{5}}\int \csc\theta\, d\theta$$

$$= \frac{1}{\sqrt{5}}\ln\left|\csc\theta-\cot\theta\right|+C$$

$\sin\theta = \frac{u}{\sqrt{5}}$



$$= \frac{1}{\sqrt{5}}\ln\left|\frac{\sqrt{5}}{u}-\frac{\sqrt{5-u^2}}{u}\right|+C$$