7.8

#28 This limit has the form $\frac{\infty}{\infty}$

$$\lim_{x\to\infty} \frac{(\ln x)^2}{x} \overset{H}{=} \lim_{x\to\infty} \frac{2(\ln x)(\frac{1}{x})}{1}$$

$$= \lim_{x\to\infty} \frac{2\ln x}{x}$$

$$\overset{H}{=} \lim_{x\to\infty} \frac{2(\frac{1}{x})}{1}$$

$$= 2(0) = \boxed{0}$$

#42 This limit has the form $0\cdot(-\infty)$

$$\lim_{x\to 0^+} \sin x \ln x = \lim_{x\to 0^+} \frac{\ln x}{\csc x}$$

$$\overset{H}{=} \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\csc x \cot x}$$

$$= -\lim_{x\to 0^+} \left(\frac{\sin x}{x} \cdot \tan x\right)$$

$$= -\left(\lim_{x\to 0^+} \frac{\sin x}{x}\right)\left(\lim_{x\to 0^+} \tan x\right)$$

$$= -1\cdot 0 = \boxed{0}$$

#54

$$y = (\tan 2x)^x$$

$$\ln y = x \cdot \ln(\tan 2x)$$

So $\lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} x \ln(\tan 2x)$

$$= \lim_{x\to 0^+} \frac{\ln(\tan 2x)}{1/x}$$

$$\overset{H}{=} \lim_{x\to 0^+} \frac{\frac{1}{\tan 2x} \cdot \sec^2 2x \cdot 2}{-\frac{1}{x^2}}$$

$$= \lim_{x\to 0^+} \frac{-2x^2 \cos 2x}{\sin 2x \cos^2 2x}$$

$$= \left(\lim_{x\to 0^+} \frac{2x}{\sin 2x}\right)\left(\lim_{x\to 0^+} \frac{-x}{\cos 2x}\right)$$

$$= 1 \cdot 0 = 0.$$

$$\Rightarrow \lim_{x\to 0^+} (\tan 2x)^x = \lim_{x\to 0^+} e^{\ln y} = e^0 = \boxed{1}$$

8.1

#14 Let $u = s$ $\quad dv = 2^s ds$

$\Rightarrow du = ds \quad v = \frac{1}{\ln 2} 2^s$

Then $\int s 2^s ds = \frac{1}{\ln 2} s 2^s - \frac{1}{\ln 2} \int 2^s ds$

$$= \boxed{\frac{1}{\ln 2} s 2^s - \frac{1}{(\ln 2)^2} 2^s + C}$$

or $\frac{2^s}{(\ln 2)^2} (s \ln 2 - 1) + C$

#22 Let $u = \ln y$ $\quad dv = \frac{1}{\sqrt{y}} dy$

$\Rightarrow du = \frac{1}{y} dy \quad v = 2\sqrt{y}$.

Then $\int_4^9 \frac{\ln y}{\sqrt{y}} dy = \left[2\sqrt{y} \ln y\right]_4^9 - \int_4^9 2 \cdot \frac{1}{\sqrt{y}} dy$

$$= (6 \ln 9 - 4 \ln 4) - \left[4\sqrt{y}\right]_4^9$$

$$= 6 \ln 9 - 4 \ln 4 - (12 - 8)$$

$$= \boxed{6 \ln 9 - 4 \ln 4 - 4}$$