HW 10

7.5 # 4, 8, 12

4. (a) $P(t) = Ce^{kt}$

$600 = P(2) = Ce^{2k}$

$75{,}000 = P(8) = Ce^{8k}$

$\frac{75{,}000}{600} = \frac{Ce^{8k}}{Ce^{2k}}$

$125 = e^{6k}$

$\ln 125 = 6k$

$k = \frac{\ln 125}{6}$

$600 = Ce^{2\left(\frac{\ln 125}{6}\right)}$

$C = 600 \cdot e^{-2\left(\frac{\ln 125}{6}\right)}$

$= 120$

$P(t) = 120\, e^{\frac{\ln 125}{6} \cdot t}$

The initial population is 120 bacteria.

(b) $P(t) = 120\, e^{\frac{\ln 125}{6} \cdot t}$

(c) $P(5) = 120\, e^{\frac{5 \ln 125}{6}} \approx 6708$

(d) $P'(5) = 6708 \cdot (\ln 125) \cdot \frac{1}{6} \approx 5398$ cells per hour

$(= P(5) \cdot k)$

(e) $200{,}000 = 120\, e^{\frac{\ln 125}{6} t}$

$1666.\overline{66} = e^{\frac{\ln 125}{6} t}$

$\left(\frac{1}{6} \ln 125\right) t = \ln 1666.\overline{66}$

$t = \frac{6 \ln 1666.\overline{66}}{\ln 125} \approx 9.22$ hours

8. (a) $P(t) = Ce^{kt}$

$800 = P(0) = Ce^{k\cdot 0} = C \qquad C = 800$

$400 = P(5.0) = 800\, e^{5k}$

$\frac{1}{2} = e^{5k}$

$\ln \frac{1}{2} = 5k \qquad k = \frac{1}{5}\ln\frac{1}{2} = -\frac{1}{5}\ln 2$

$P(t) = 800\, e^{(-\frac{1}{5}\ln 2)t}$

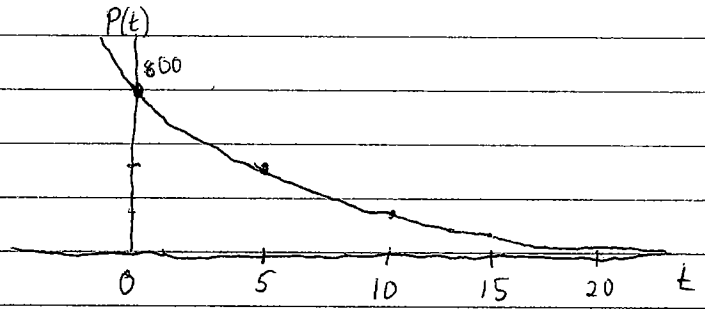
(b) $P(30) = 800\, e^{(-\frac{1}{5}\ln 2)\cdot 30} = 12.5$ mg

(c) $1 = 800\, e^{(-\frac{1}{5}\ln 2)t}$

$\frac{1}{800} = e^{(-\frac{1}{5}\ln 2)t}$

$\ln\frac{1}{800} = (-\frac{1}{5}\ln 2)t \qquad t = 5\cdot\frac{\ln 800}{\ln 2} \approx 48.2$ days

(d)



12. $\frac{dy}{dx} = 2y$ i.e. $k = 2$

$$\boxed{y = 5e^{2x}}$$