

Section 5.2.

(P₁)

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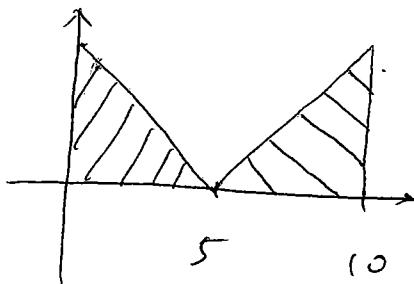
(a). $\int_0^2 g(x) = \frac{1}{2} \cdot 4 \cdot 2 = \boxed{4}$ (area of a triangle)

(b). $\int_2^6 g(x) = -\frac{1}{2} \pi (2)^2 = \boxed{-2\pi}$ (negative of the area of a semicircle)

(c). $\int_6^7 g(x) = \frac{1}{2} \cdot 1 \cdot 1 = \frac{1}{2}$ (area of a triangle)

$$\begin{aligned} \int_0^7 g(x) dx &= \int_0^2 g(x) dx + \int_2^6 g(x) dx + \int_6^7 g(x) dx \\ &= 4 + (-2\pi) + \frac{1}{2} \\ &= \boxed{4.5 - 2\pi} \end{aligned}$$

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$\int_0^{10} |x-5| dx$ can be interpreted as the sum of the area of the two shaded triangles.

So $\int_0^{10} |x-5| dx = 2 \cdot \left(\frac{1}{2}\right) \cdot 5 \cdot 5 = \boxed{25}$

Section 5.3

P₂

#2

$$(a). \quad g(x) = \int_0^x f(t) dt.$$

$$\text{So } g(0) = \int_0^0 f(t) dt = \boxed{0}$$

$$g(1) = \int_0^1 f(t) dt = \frac{1}{2} \cdot 1 \cdot 1 = \boxed{\frac{1}{2}} \quad (\text{area of triangle})$$

$$\begin{aligned} g(2) &= \int_0^2 f(t) dt = \int_0^1 f(t) dt + \int_1^2 f(t) dt \\ &= \frac{1}{2} + (-\frac{1}{2} \cdot 1 \cdot 1) = 0 \end{aligned}$$

$$\begin{aligned} g(3) &= g(2) + \int_2^3 f(t) dt \\ &= 0 - \frac{1}{2} \cdot 1 \cdot 1 = \boxed{-\frac{1}{2}} \end{aligned}$$

$$\begin{aligned} g(4) &= g(3) + \int_3^4 f(t) dt \\ &= -\frac{1}{2} + \frac{1}{2} \cdot 1 \cdot 1 = \boxed{0} \end{aligned}$$

$$\begin{aligned} g(5) &= g(4) + \int_4^5 f(t) dt \\ &= 0 + 1.5 = \boxed{1.5} \end{aligned}$$

$$\begin{aligned} g(6) &= g(5) + \int_5^6 f(t) dt \\ &= 1.5 + 2.5 = \boxed{4} \end{aligned}$$

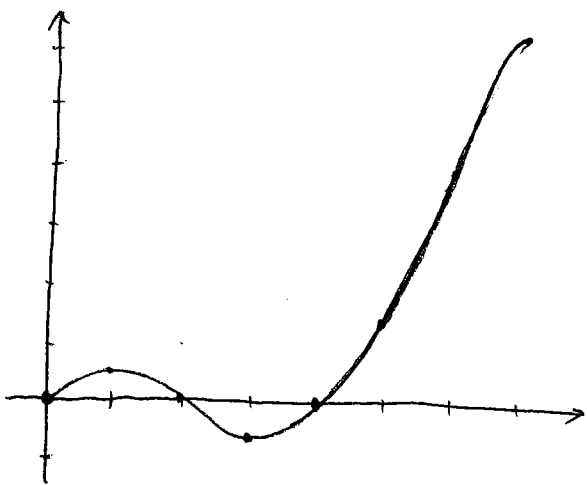
(b) $g(7) = g(6) + \int_6^7 f(t) dt \approx 4 + 2.2 = \boxed{6.2}$

(c) The answers from part (a) and part (b) show that

$$g_{\max} = g(7) = 6.2$$

$$g_{\min} = g(3) = -\frac{1}{2}$$

(d)



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$$G(x) = \int_x^1 \cos \sqrt{t} dt = - \int_1^x \cos \sqrt{t} dt.$$

$$\Rightarrow G'(x) = - \frac{d}{dx} \int_1^x \cos \sqrt{t} dt = \boxed{-\cos \sqrt{x}}$$

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$$\int_1^8 \sqrt[3]{x} dx = \int_1^8 x^{\frac{1}{3}} dx = \left[\frac{3}{4} x^{\frac{4}{3}} \right]_1^8$$

$$= \frac{3}{4} (8^{\frac{4}{3}} - 1^{\frac{4}{3}}) = \frac{3}{4} (2^4 - 1) = \frac{3}{4} \cdot 15 = \boxed{\frac{45}{4}}$$