1. Find the absolute maximum and the absolute minimum of the function $f(x) = 4x^2 - 12x + 10$ on the interval $[1, 2]$. Closed Interval Method:

$f'(x) = 8x - 12$

$8x - 12 = 0$

$8x = 12$

$\underline{x = 3/2}$, one critical number.

evaluate f at critical numbers and end points:

$f(1) = 4 - 12 + 10 = 2$

$f(3/2) = 4\left(\frac{9}{4}\right) - 12\left(\frac{3}{2}\right) + 10 = 1$

$f(2) = 4(4) - 12(2) + 10 = 2.$

Absolute minimum is 1.
Absolute maximum is 2.

2(a) Find the linearization of $f(x) = \dfrac{1}{\sqrt{1-x}}$ at $a = 0$. Then, estimate $\dfrac{1}{\sqrt{0.99}}$.

$f(x) = (1-x)^{-1/2}$, $f'(x) = -\frac{1}{2}(1-x)^{-3/2} \cdot (-1)$

$= \dfrac{1}{2(1-x)^{3/2}}$.

$f'(0) = \frac{1}{2}$. So, linearization is

$\boxed{L(x) = \underbrace{1}_{f(0)} + \frac{1}{2}x}$.

Next, $\dfrac{1}{\sqrt{.99}}$ is $f(.01) \approx L(.01)$

$= 1 + \frac{1}{2}(.01)$

$= \boxed{1.005}$

(b) Let $f(x) = 3x^2 - 2x + 5$. What, specifically, does the Mean Value Theorem tell us about this function over the interval $[0, 2]$?

First, $f(0) = 5$, $f(2) = 3(4) - 2(2) + 5 = 13$.

The MVT says that there is a number c in the interval $[0, 2]$ such that $f'(c) = \dfrac{13-5}{2} = 4$.

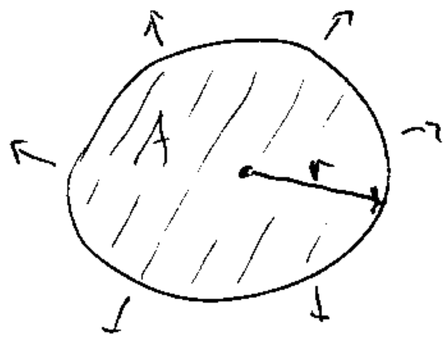
3. Oil spilled from a ruptured tanker spreads in a circle whose area increases at a constant rate of 6 mi^2/h.

(a) Let r be the raduis of the oil spill and A its area. Draw a picture and write an equation relating r and A.

(b) Find an equation involving $r'(t)$ and $A'(t)$.

(c) How fast is the radius of the spill increasing when the area is 9 mi^2[redacted]? [Don't forget to include units in your answer.]

(a)



$$\boxed{A = \pi r^2}$$

(b) $$\frac{d}{dt}(A) = \frac{d}{dt}(\pi r^2)$$

$$\boxed{A'(t) = \pi \cdot 2r(t)\,r'(t)}$$

(3) When $A = 9$, $9 = \pi r^2$

$$r^2 = \frac{9}{\pi}$$

$$r = \frac{3}{\sqrt{\pi}} \quad \left(\frac{-3}{\sqrt{\pi}} \text{ can't be the radius}\right)$$

Also, $A' = 6$.

$$\cancel{6} = \pi \cdot \cancel{2}\left(\frac{\cancel{3}}{\sqrt{\pi}}\right) r'(t)$$

$$r' = \frac{\sqrt{\pi}}{\pi} = \frac{1}{\sqrt{\pi}}.$$

The radius is increasing at a rate of $\frac{1}{\sqrt{\pi}}$ mi/h.

4. Consider the function $f(x) = 3x^5 - 5x^3$.

(a) Does f have any symmetry? (Is it even, or odd, or neither?)

(b) Find the intervals on which f is increasing, and on which f is decreasing.

(c) Find the local maxima and minima of f.

(d) Find the intervals on which the graph of f is concave up, and on which the graph is concave down.

(a) $f(-x) = 3(-x)^5 - 5(-x)^3 = -3x^5 + 5x^3$
$= -(3x^5 - 5x^3)$
$= -f(x)$.

$\boxed{f \text{ is odd (and not even)}}$.

(b) $f'(x) = 15x^4 - 15x^2 = 15x^2(x+1)(x-1)$

critical #s: $x = 0, 1, -1$.

$f' > 0$, $f' < 0$, $f' < 0$, $f' > 0$

$-1 \quad 0 \quad 1$

$\boxed{\begin{array}{l} \text{Increasing on } (-\infty, -1) \cup (1, \infty) \\ \text{Decreasing on } (-1, 0) \cup (0, 1) \end{array}}$

(c) $\boxed{\begin{array}{l} \text{local max at } x = -1 \\ \text{local min at } x = 1 \end{array}}$ (First derivative test)

(d) $f''(x) = 60x^3 - 30x = 30x(2x^2 - 1)$

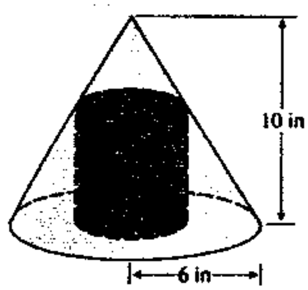
$x = 0$, $x = \pm\sqrt{\tfrac{1}{2}}$

$f'' < 0$, $f'' > 0$, $f'' < 0$, $f'' > 0$

$-\sqrt{\tfrac{1}{2}} \quad 0 \quad \sqrt{\tfrac{1}{2}}$

$\boxed{\begin{array}{l} \text{Concave up on } (-\sqrt{\tfrac{1}{2}}, 0) \cup (\sqrt{\tfrac{1}{2}}, \infty) \\ \text{Concave down on } (-\infty, -\sqrt{\tfrac{1}{2}}) \cup (0, \sqrt{\tfrac{1}{2}}) \end{array}}$

5. We want to find the radius and height of the cylinder of largest volume that can be inscribed in a cone with radius 6 inches and height 10 inches.

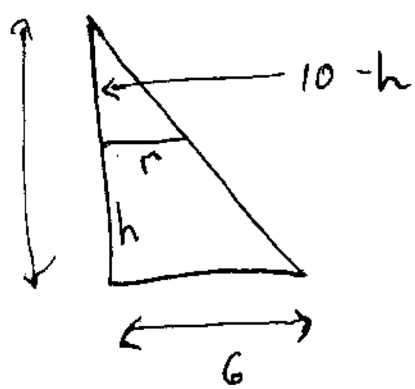


(a) Find a constraint relating r and h. [Hint: use similar triangles.]

(b) Express the volume in terms of one variable, and give the domain of this function.

(c) Use an appropriate method to solve the problem.

(a)



Similar triangles $\Rightarrow$

$$\boxed{\frac{10}{6} = \frac{10-h}{r}}$$

(b) $V = \pi r^2 h$ use $r = \frac{3}{5}(10-h)$:

$$\boxed{V(h) = \pi \cdot \frac{9}{25} \cdot (10-h)^2 h \quad \text{with domain } 0 \le h \le 10}$$

<u>closed interval problem.</u>

(c) $V(h) = \frac{9\pi}{25}(100h - 20h^2 + h^3)$

$V'(h) = \frac{9\pi}{25}(100 - 40h + 3h^2) = 0$

$h = \frac{40 \pm \sqrt{1600 - 1200}}{6} = \frac{40 \pm 20}{6}$

$= 10, \frac{10}{3}.$

Compare $V(0), V(\frac{10}{3}), V(10)$.

these are zero — so max Volume is $V(\frac{10}{3})$.

When $h = \frac{10}{3}$, $r = \frac{3}{5}(10 - \frac{10}{3}) = 4$. So $\boxed{r = 4 \text{ inches}, h = \frac{10}{3} \text{ inches}}$