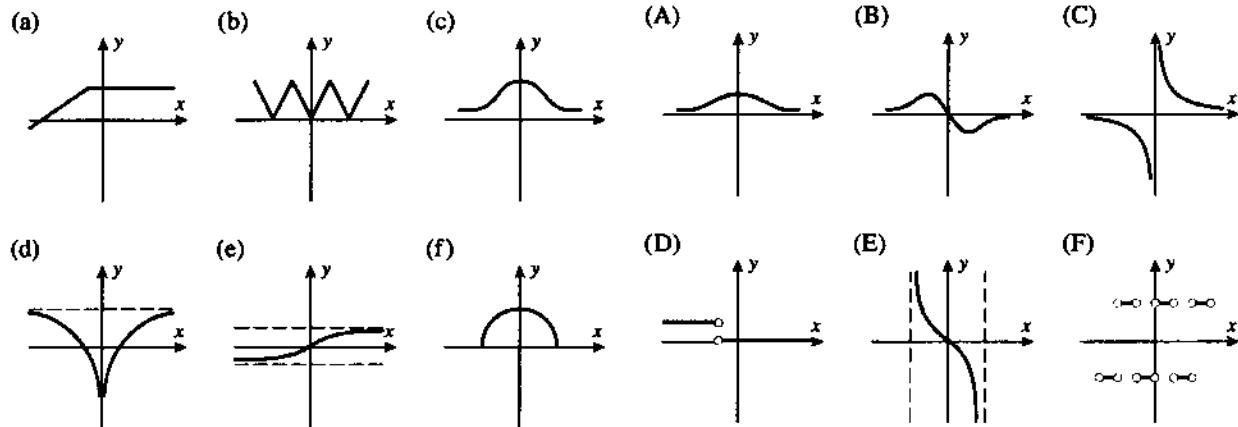
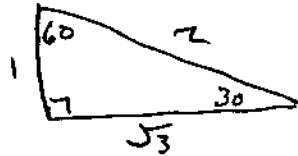
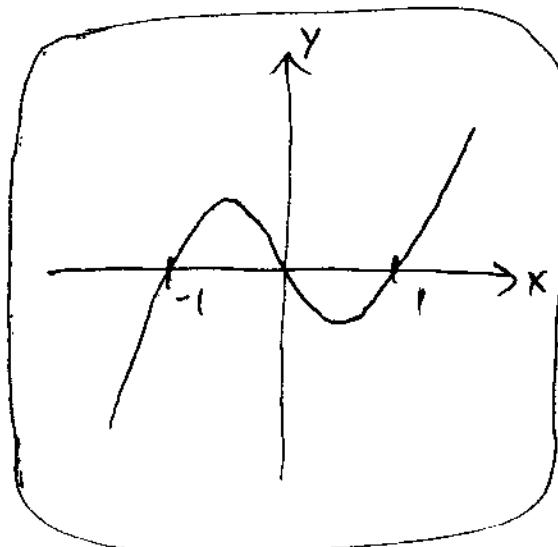
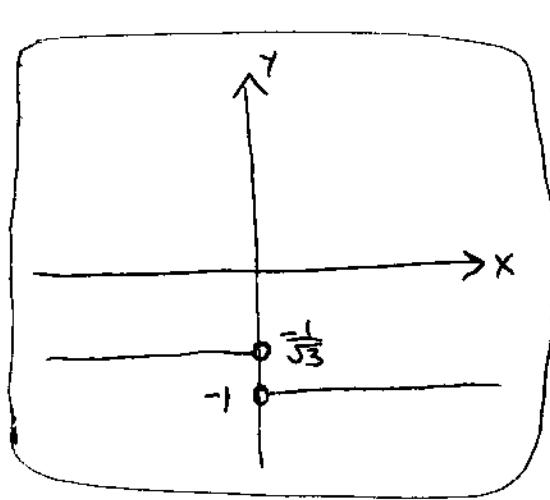
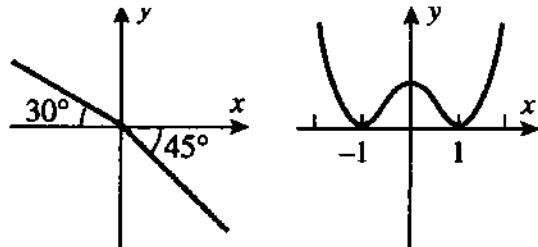


1(a) Match the graphs of the functions shown in (a)-(f) with the graphs of their derivatives in (A)-(F).



(a)	D	(b)	F	(c)	B
(d)	C	(e)	A	(f)	E

1(b) Sketch the graph of the derivative of each function shown below:



2(a) Explain why the function

$$f(x) = \begin{cases} 3x - 1 & x \leq 1 \\ x^2 & x > 1 \end{cases}$$

is not continuous at  $x = 1$ . [It may be helpful to draw a graph.]

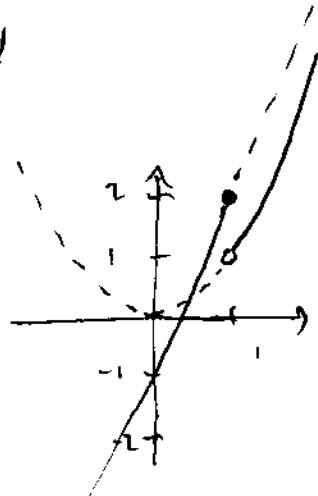
cont. at 1 means:  $\lim_{x \rightarrow 1} f(x) = f(1)$

but  $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} 3x - 1 = 3(1) - 1 = 2$

and  $\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} x^2 = 1^2 = 1$

Hence  $\lim_{x \rightarrow 1} f(x)$  does not exist.

So  $f$  is not continuous at 1.



(b) Find a constant  $k$  that makes the function

$$g(x) = \begin{cases} 3x - 1 & x \leq 1 \\ kx^2 & x > 1 \end{cases}$$

continuous at  $x = 1$ .

$g(1) = 2$  and  $\lim_{x \rightarrow 1^-} g(x) = \lim_{x \rightarrow 1^-} 3x - 1 = 2$

$\lim_{x \rightarrow 1^+} g(x) = \lim_{x \rightarrow 1^+} kx^2 = k$

If  $\boxed{k=2}$  then  $\lim_{x \rightarrow 1} g(x) = 2 = g(1)$

and  $g$  is continuous at 1.

3(a) Use implicit differentiation to find  $\frac{dy}{dx}$  for the curve  $y^3 + yx^2 + x^2 - 3y^2 = 0$ .

(b) Find an equation of the tangent line to the curve at the point  $(0, 3)$ .

$$(a) \frac{d}{dx} (y^3 + yx^2 + x^2 - 3y^2) = \frac{d}{dx}(0)$$

$$3y^2 \frac{dy}{dx} + (y(2x) + \frac{dy}{dx}x^2) + 2x - 6y \frac{dy}{dx} = 0$$

(chain rule!)

$$\frac{dy}{dx}(3y^2 + x^2 - 6y) + 2x(y+1) = 0$$

$$\boxed{\frac{dy}{dx} = \frac{-2x(y+1)}{3y^2 + x^2 - 6y}}$$

$$(b) \text{ Slope at } (0, 3) \text{ is } \frac{2(0)(3+1)}{3(3)^2 + (0)^2 - 6(3)} = 0$$

Tangent line is

$$(y-3) = 0(x-3)$$

$$\boxed{y=3}$$

(slope 0, goes through  $(0, 3)$ )

4. Calculate each derivative  $\frac{dy}{dx}$ .

(a)  $y = (x^2 + x)(x^2 - x)$

$$\frac{dy}{dx} = \boxed{(x^2 + x)(2x - 1) + (2x + 1)(x^2 - x)}$$

or write  $y = x^4 - x^2$ ,  $\frac{dy}{dx} = \boxed{4x^3 - 2x}$

(they're the same)

(b)  $y = \frac{(x^2 + 1)}{\sec x}$

$$\frac{dy}{dx} = \boxed{\frac{(\sec x)(2x) - (x^2 + 1)\sec x \tan x}{\sec^2 x}}$$

(c)  $y = \cos(\cos x)$

$$\frac{dy}{dx} = \boxed{-\sin(\cos x)(-\sin x)}$$

(d)  $y = \cos^2(3\sqrt{x})$

$$\frac{dy}{dx} = 2\cos(3\sqrt{x}) \frac{d}{dx}(\cos(3\sqrt{x}))$$

$$= 2\cos(3\sqrt{x})(-\sin(3\sqrt{x}) \frac{d}{dx}(3\sqrt{x}))$$

$$= \boxed{2\cos(3\sqrt{x})(-\sin(3\sqrt{x})) \frac{3}{2\sqrt{x}}}$$

5(a) Here is some information about a function  $f(x)$ :

$x$	$f(x)$	$f'(x)$
2	1	7
8	5	-3

Define  $g(x) = (f(x))^3$  and  $h(x) = f(x^3)$ . Use the Chain Rule to calculate  $g'(2)$  and  $h'(2)$ .

$$g'(x) = 3(f(x))^2 f'(x)$$

$$g'(2) = 3(f(2))^2 f'(2) = 3(1)^2 \cdot 7 = \boxed{21}$$

$$h'(x) = f'(x^3) \cdot 3x^2$$

$$h'(2) = f'(8) \cdot 3(2)^2 = (-3) \cdot 12 = \boxed{-36}$$

(b) Find  $\lim_{x \rightarrow 0} \frac{\tan x}{x}$ , and include (briefly) your reasoning.

$$\lim_{x \rightarrow 0} \frac{\tan x}{x} = \lim_{x \rightarrow 0} \frac{\sin x}{x \cos x} = \lim_{x \rightarrow 0} \left( \frac{\sin x}{x} \right) \left( \frac{1}{\cos x} \right)$$

$$\lim_{x \rightarrow 0} \tan x = \left( \lim_{x \rightarrow 0} \frac{\sin x}{x} \right) \left( \lim_{x \rightarrow 0} \frac{1}{\cos x} \right) \text{ if these limits exist}$$

$\frac{1}{\cos(\alpha)}$  (direct subs b/c  $\cos x$  is continuous)

So these units exist and are both 1.

$$\Rightarrow \boxed{\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1}$$