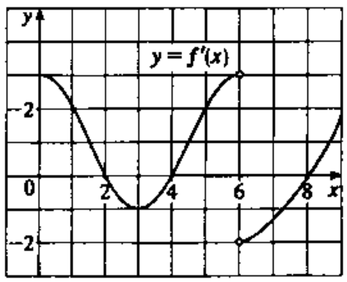
1(a) The graph of the derivative f' of a continuous function f is shown.

(i) On what intervals is f increasing? Decreasing?

(ii) At what values of x does f have a local maximum? Local minimum?

(iii) On what intervals is f concave up? Concave down?

(iv) State the x-coordinate(s) of the point(s) of inflection.



(i) increasing on $(0,2)$ and $(4,6)$ and $(8,9)$
decreasing on $(2,4)$ and $(6,8)$

(ii) local min. at $x = 4, 8$
local max. at $x = 2, 6$

(iii) concave up on $(3,6)$ and $(6,9)$
concave down on $(0,3)$

(iv) inflection point at $x = 3$

(b) Suppose that $h(x) = f(x)g(x)$ and $F(x) = f(g(x))$, where $f(2) = 3$, $g(2) = 5$, $g'(2) = 4$, $f'(2) = -2$, and $f'(5) = 11$.

(i) Find $h'(2)$.

(ii) Find $F'(2)$.

(i) $h'(x) = f'(x)g(x) + f(x)g'(x)$

$h'(2) = (-2)(5) + (3)(4) = \boxed{2}$

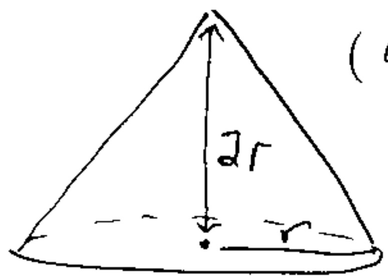
(ii) $F'(x) = f'(g(x))g'(x)$

$F'(2) = f'(2) \cdot 4 = \boxed{44}$

2. Wheat is poured through a chute at a rate of 10 ft^3/min, and falls in a cone-shaped pile whose bottom radius is always half the height.

(a) How fast will the radius of the base be increasing when the pile is 8 ft high?

(b) How fast will the circumference of the base be increasing when the pile is 8 ft high?



(a) $V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi r^2 (2r)$

$\underline{V = \frac{2\pi}{3} r^3}$

$\underline{\frac{dV}{dt} = \frac{2\pi}{3} \cdot 3r^2 \cdot \frac{dr}{dt}}$ note, $h = 8 \Rightarrow r = 4$.

$10 = 2\pi(16)\frac{dr}{dt}$

$\frac{dr}{dt} = \boxed{\frac{10}{32\pi}\ \text{ft./min.}}$

(b) $\underline{C = 2\pi r}$

$\underline{\frac{dC}{dt} = 2\pi \frac{dr}{dt}}$ and $\frac{dr}{dt} = \frac{10}{32\pi}$

$\frac{dC}{dt} = \frac{20}{32} = \boxed{\frac{5}{8}\ \text{ft/min.}}$

3(a) Use implicit differentiation to find the slope of the tangent line to the curve $x^3 + y^3 = 3xy$ at the point $(\frac{3}{2}, \frac{3}{2})$.

$$\frac{d}{dx}(x^3 + y^3) = \frac{d}{dx}(3xy)$$

$$3x^2 + 3y^2\frac{dy}{dx} = 3x\frac{dy}{dx} + 3y \quad \left(\begin{matrix}\text{product} \\ \text{rule}\end{matrix}\right)$$

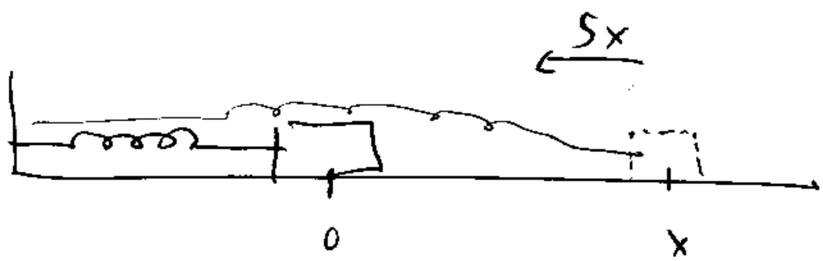
$$x^2 + y^2 y' = xy' + y$$

$$x^2 - y = (x - y^2)y' \implies \frac{dy}{dx} = \frac{x^2 - y}{x - y^2}$$

At $(3/2, 3/2)$, slope is

$$\frac{dy}{dx} = \frac{(\frac{3}{2})^2 - (\frac{3}{2})}{(\frac{3}{2}) - (\frac{3}{2})^2} = \boxed{-1}$$

3(b) A block on the floor is attached to the wall by a spring. The spring exerts a force of $5x$ lbs when it is stretched a distance of x feet from its rest position. Find the amount of work done against the spring when the block is pulled from its rest position a distance of 6 feet.



$$\text{Work} = \text{limit of } \sum_{i=1}^{n} \underbrace{5x}_{\text{force}}\ \underbrace{\Delta x}_{\text{distance}}$$

$$= \int_0^6 5x\,dx$$

$$= \frac{5}{2}x^2 \Big|_0^6 = \frac{5}{2}(36) = \boxed{90 \text{ ft-lbs}}$$

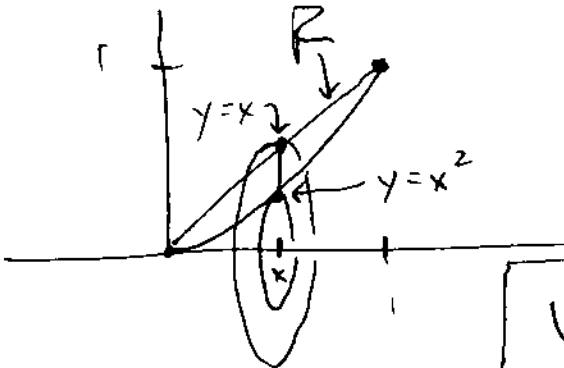
4. Let R be the region bounded by the curves $y = x$ and $y = x^2$. Consider the solid of revolution obtained by rotating R about the x-axis.

(a) Write down an integral giving the volume of the solid using rings. Draw a picture of the region, and a typical ring.

(b) Write down an integral giving the volume using cylindrical shells. Draw a picture of a typical shell.

(c) Calculate the volume, using either integral.

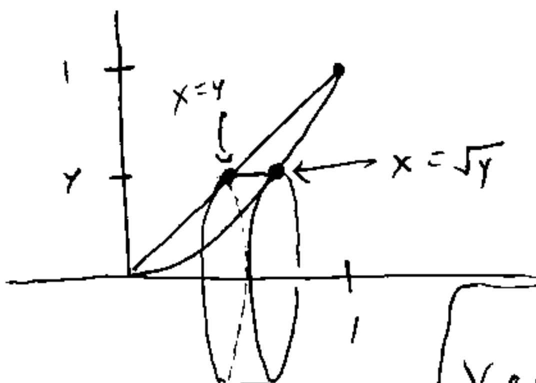
(a)



$$A(x) = \pi(x)^2 - \pi(x^2)^2$$

$$\boxed{Vol = \int_0^1 \pi(x^2 - x^4)\,dx}$$

(b)



$$A(y) = 2\pi y(\sqrt{y} - y)$$

$$\boxed{Vol = \int_0^1 2\pi y(\sqrt{y} - y)\,dy}$$

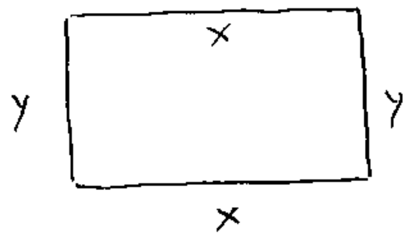
(c) $$\int_0^1 \pi(x^2 - x^4)\,dx = \frac{\pi}{3}x^3 - \frac{\pi}{5}x^5 \Big|_0^1 = \frac{\pi}{3} - \frac{\pi}{5} = \boxed{\frac{2\pi}{15}}$$

OR

$$\int_0^1 (2\pi y^{3/2} - 2\pi y^2)\,dy = 2\pi \cdot \frac{2}{5} y^{5/2} - 2\pi \cdot \frac{1}{3} y^3 \Big|_0^1$$

$$= \frac{4\pi}{5} - \frac{2\pi}{3} = \boxed{\frac{2\pi}{15}}$$

5. A rectangular area of 3200 ft^2 is to be enclosed by a fence. Two opposite sides will use fencing costing \$1 per foot and the remaining sides will use fencing costing \$2 per foot. Find the dimensions of the rectangle having the smallest cost.



x at \$1 per foot
y at \$2 per foot

Cost $= 2x + 4y$

constraint: $3200 = xy \Rightarrow y = \frac{3200}{x}$

$$\text{Cost}(x) = 2x + 4\left(\frac{3200}{x}\right) = 2x + \frac{12800}{x}.$$

minimize this over $0 < x < \infty$:

$$\text{Cost}'(x) = 2 - \frac{12800}{x^2} = 0$$

$$2x^2 = 12800$$
$$x^2 = 6400$$
$$x = \pm 80$$
$$\underline{x = 80} \quad \text{since } x > 0$$

note, Cost$'(x)$ is negative for $x < 80$, positive for $x > 80$. Hence, minimum occurs at $x = 80$.

Next $y = \frac{3200}{80} = 40$.

Dimensions: $\boxed{\text{80 ft. of \$1 fence} \times \text{40 ft. of \$2 fence}}$

6. Compute the following:

(a) $\int t^4 \sqrt[3]{3-5t^5}\,dt$

$u = 3-5t^5$, $du = -25t^4\,dt$

so $\frac{-1}{25}\,du = t^4\,dt$

$$\| $$

$$\int \frac{-1}{25} u^{\frac{1}{3}}\,du = \frac{-1}{25}\cdot\frac{3}{4}u^{4/3} + C$$

$$= \boxed{\frac{-3}{100}(3-5t^5)^{4/3} + C}$$

(b) $\lim_{x\to\infty} \dfrac{x^2-4x+12}{3x^2+3x+4}$

$$= \lim_{x\to\infty} \frac{1 - \frac{4}{x} + \frac{12}{x^2}}{3 + \frac{3}{x} + \frac{4}{x^2}}$$

$$= \frac{\lim_{x\to\infty} 1 - \lim_{x\to\infty}\frac{4}{x} + \lim_{x\to\infty}\frac{12}{x^2}}{\lim_{x\to\infty} 3 + \lim_{x\to\infty}\frac{3}{x} + \lim_{x\to\infty}\frac{4}{x^2}}$$

$$= \boxed{\frac{1}{3}}$$

(c) The average value of $f(x) = 4\sin x\cos x$ on the interval $[0, \pi/4]$.

$$\text{Avg} = \frac{1}{\pi/4 - 0}\int_0^{\pi/4} 4\sin x\cos x\,dx$$

$u = \sin x$
$du = \cos x\,dx$

$$= \frac{4}{\pi}\int_0^{\frac{\sqrt{2}}{2}} 4u\,du = \frac{4}{\pi}(2u^2)\Big|_0^{\frac{\sqrt{2}}{2}}$$

$$= \frac{4}{\pi}\cdot 2\left(\frac{2}{4}\right) = \boxed{\frac{4}{\pi}}$$