Solution to Some problems of HW 9:

Section 3.5:

6- Use the guidelines of this section to sketch the curve.

$y = x^5 - 5x$

A- Domain $= \mathbb{R}$

B- Intercepts:

- x-intercept: Solve $x^5 - 5x = 0$

$$x(x^4 - 5) = 0$$

$$x = 0 \quad \text{or} \quad x^4 - 5 = 0$$

$$x^2 = \sqrt{5} \quad \text{and} \quad x = \pm \sqrt[4]{5}$$

- y-intercept: Set $x = 0$,

$$y = 0$$

Hence, we get $(0,0)$, $(\sqrt[4]{5}, 0)$ $(-\sqrt[4]{5}, 0)$

C. Symmetry:

$f(-x) = (-x)^5 - 5(-x) = -x^5 + 5x = -(x^5 - 5x) = -f(x)$

So, f is odd and hence we have symmetry about the origin.

D. Asymptotes:

No asymptotes.

E- Intervals of Increase / Decrease:

(1) $f'(x) = 5x^4 - 5$

(2) Critical Numbers:

Solve $f'(x) = 0$

$$5(x^4 - 1) = 0$$

$$x^2 = 1 \quad \& \quad x = \pm 1$$

		-1		1	
$f'(x)$	$+$	0	$-$	0	$+$
$f(x)$	$\nearrow$		$\searrow$		$\nearrow$

Local max (at -1) Local min (at 1)

Increasing on $(-\infty, -1) \cup (1, \infty)$

Decreasing on $(-1, 1)$

F. Local Max/Min:

Local maximum value $f(-1) = 4$.

Local minimum value $f(1) = -4$.

G. Concavity:

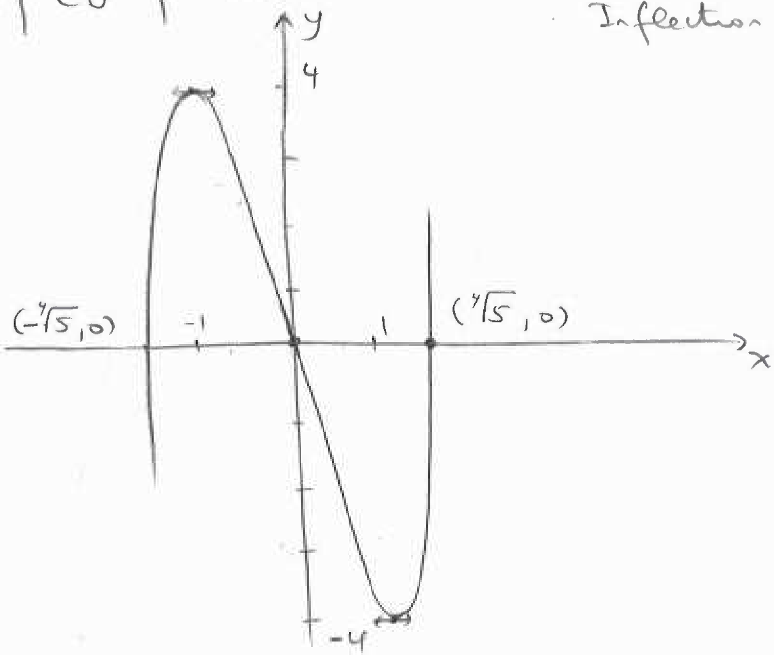
(1) $f''(x) = 20x^3$

(2) Solve $f''(x) = 0$ to get $20x^3 = 0$ and $x = 0$

	0	
$f''(x)$	$-$	0	$+$
$f(x)$	CD		CU

- Concave up on $(0, \infty)$
- Concave down on $(-\infty, 0)$
- Inflection point: $(0, 0)$.

H.



18. Use the guidelines of this section to sketch the curve.

$$y = \frac{x}{x^3-1}.$$

A. Domain:

$x^3 - 1 \neq 0$, so $x \neq 1$

Domain $= (-\infty, 1) \cup (1, \infty)$

B. Intercepts:

x-intercept: Solve $\frac{x}{x^3-1} = 0$ i.e. $x = 0$

y-intercept: Plug in $x=0$ to get $y=0$

So we have $(0,0)$.

C. Symmetry:

No symmetry

D. Assymptotes:

(1) H.A:

$$\lim_{x\to\pm\infty} f(x) = \lim_{x\to\pm\infty} \frac{x}{x^3-1} = \lim_{x\to\pm\infty} \frac{\frac{1}{x^2}}{1-\frac{1}{x^3}} = 0$$

So, $y=0$ is a H.A.

(2) V.A:

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \frac{x}{x^3-1} = -\infty$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \frac{x}{x^3-1} = +\infty$$

So $x=1$ is a V.A.

E. Intervals of Increase | Decrease:

(1) $$f'(x) = \frac{(1)(x^3-1) - (3x^2)(x)}{(x^3-1)^2} = \frac{x^3-1-3x^3}{(x^3-1)^2} = \frac{-2x^3-1}{(x^3-1)^2}$$

(2) Solve $f'(x) = 0$ to get

$-2x^3 - 1 = 0$

$x^3 = -\frac{1}{2}$ and $x = -\sqrt[3]{\frac{1}{2}}$

		$-\sqrt[3]{\frac{1}{2}}$		1	
$f'(x)$	$+$	0	$-$		$-$
$f(x)$	↗		↘		↘

V.A.

Local max.

Increasing on $(-\infty, -\sqrt[3]{\frac{1}{2}})$
Decreasing on $(-\sqrt[3]{\frac{1}{2}}, 1) \cup (1, \infty)$

F. Local Min / Max:

Local max. value is $f(-\sqrt[3]{\frac{1}{2}}) = \frac{2}{3}\sqrt[3]{\frac{1}{2}}$.

No local min.

G. Concavity:

(1) $$f''(x) = \frac{(-6x^2)(x^3-1)^2 - 2(x^3-1)\cdot(3x^2)(-2x^3-1)}{(x^3-1)^4}$$

$$= \frac{-6x^2(x^3-1)[x^3-1+(-2x^3-1)]}{(x^3-1)^4} = \frac{-6x^2(x^3-1)(-x^3-2)}{(x^3-1)^4}$$

$$= \frac{6x^2(x^3+2)}{(x^3-1)^3}$$

(2) Solve $f''(x)=0$

$6x^2(x^3+2)=0$

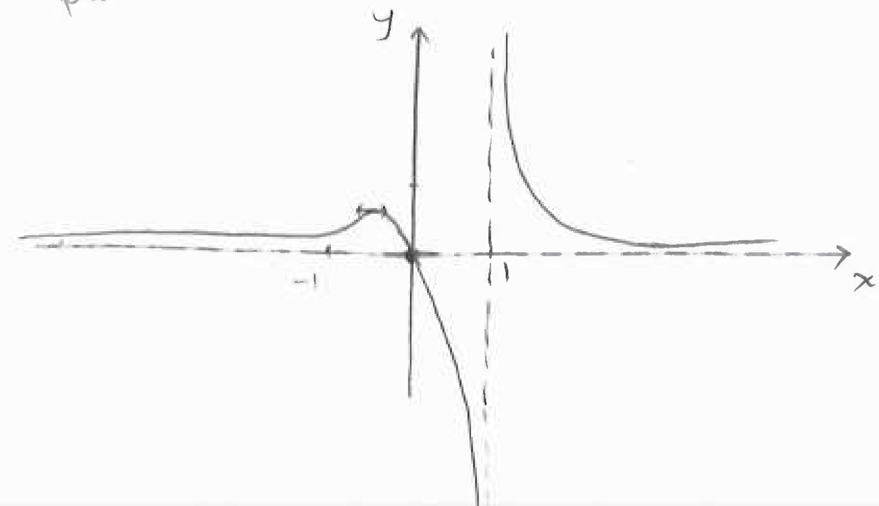
$x=0$ or $x=-\sqrt[3]{2}$

		$-\sqrt[3]{2}$		0		1	
$f''(x)$	$+$	0	$-$	0	$-$		$+$
$f(x)$	CU		CD		CD		CU

Inflection pt.

Concave up on $(-\infty, -\sqrt[3]{2}) \cup (1, \infty)$
Concave down on $(-\sqrt[3]{2}, 1)$
Inflection pt. $(-\sqrt[3]{2}, \frac{1}{3}\sqrt[3]{2})$

H.



46. Find an equation of the slant asymptote. Do not sketch the curve.

$$f(x) = y = \frac{2x^3+x^2+x+3}{x^2+2x}$$

(1) Apply long division:

$$\begin{array}{r|l} & 2x - 3 \\ \hline \boxed{x^2}+2x & \boxed{2x^3}+x^2+x+3 \\ \ominus & 2x^3+4x^2 \\ \hline & \boxed{-3x^2}+x+3 \\ \ominus & -3x^2-6x \\ \hline & 7x+3 \end{array}$$

So, $y = \dfrac{2x^3+x^2+x+3}{x^2+2x}$

$= (2x-3) + \dfrac{7x+3}{x^2+2x}$.

(2) Show: $\lim_{x \to \pm\infty} [f(x) - (2x-3)] = 0$

$$\lim_{x \to \pm\infty} (f(x) - (2x-3)] = \lim_{x \to \pm\infty} \left(\frac{7x+3}{x^2+2x}\right) = \lim_{x \to \pm\infty} \frac{\frac{7}{x}+\frac{3}{x^2}}{1+\frac{2}{x}} = 0$$

So, $y = 2x-3$ is a slant asymptote of $f(x) = \dfrac{2x^3+x^2+x+3}{x^2+2x}$.

Section 3.7:

2- Find two numbers whose difference is 100 and whose product is a minimum.

Let x and y be the two numbers.

(1) Then, $x - y = 100$ and $\boxed{y = x - 100}$

(2) Let P be their product, then

$$P = x \cdot y = x(x-100) = x^2 - 100x.$$

(3) To minimize their product:

$$P' = 2x - 100$$

Solve $P' = 0$

$$2x - 100 = 0 \quad \text{and} \quad x = 50$$

To check whether we have abs. max or min at $x = 50$, use the 2^{nd} derivative test: $P'' = 2 > 0$. and we have an abs. min at $x = 50$

So, the numbers are:

$$x = 50 \quad \text{and} \quad y = 50 - 100 = -50$$

i.e. 50 and -50.

8- Find the dimensions of a rectangle with area 00 m^2 whose perimeter is as small as possible.

Let l be the length and w be the width of the rectangle

(1) Then, $l \cdot w = 1000$

(2) Let P be the perimeter, so $P = 2l + 2w$

(3) To minimize P,

. first solve for w in terms of l: $l \cdot w = 1000$, $w = \dfrac{1000}{l}$

. $P = 2l + 2w = 2l + \dfrac{2000}{l} = \dfrac{2l^2 + 2000}{l}$

. $P' = 2 - \dfrac{2000}{l^2}$

Solve $P' = 0$

$2 - \dfrac{2000}{l^2} = 0$ $\quad \dfrac{2000}{l^2} = 2$ $\quad l^2 = 1000$ and $l = 10\sqrt{10}$ m

To check that we have an absolute minimum at $l = 10\sqrt{10}$ m

$$P'' = \frac{4{,}000}{l^3}$$

and $P''(10\sqrt{10}) = \frac{4000}{10^4\sqrt{10}} > 0$.

So we have an absolute minimum at $l = 10\sqrt{10}$ m

$$w = \frac{1000}{l} = \frac{1000}{10\sqrt{10}} = \frac{100}{\sqrt{10}} = 10\sqrt{10} \text{ m}$$

Hence, length = width = $10\sqrt{10}$ m (Rectangle is actually a square).