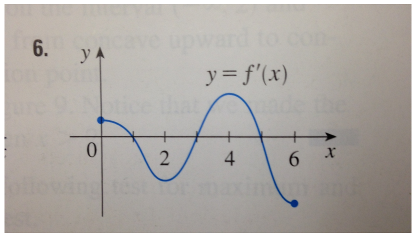
Week 8 Homework (Answers from Stewart's Solution Manuel)

3.3

6. The graph of the derivative f' of a function is shown.



(a) On what intervals is f increasing or decreasing?
(b) At what values of x does f has a local maximum or minimum?

(a) Increasing on $(0,1)$ and $(3,5)$; Decreasing on $(1,3)$ and $(5,6)$.
(b) Local Maxima at x=1 and x=5 (positive to negative)
Local Minima at x=3 (negative to positive)

12. $f(x)=\dfrac{x}{x^2+1}$
(a) Find the intervals on which f is increasing or decreasing.
(b) Find the local maximum and minimum values of f.
(c) Find the intervals of concavity and the inflection points.

(a) $f'(x)=\dfrac{(x^2+1)(1)-x(2x)}{(x^2+1)^2}=-\dfrac{(x+1)(x-1)}{(x^2+1)^2}$.
So f is increasing on $(-1,1)$ and decreasing on $(-\infty,-1)$ and $(1,\infty)$.

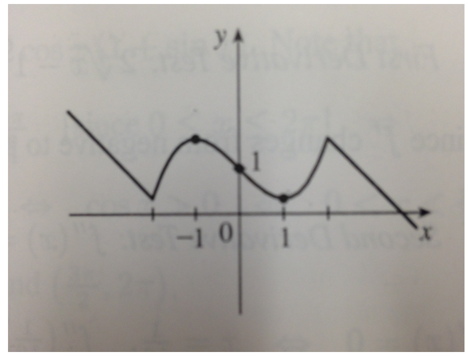
(b) $f(-1)=-1/2$ **is a local minimum (decreasing to increasing), and $f(1)=1/2$ is a local maximum (increasing to decreasing).**

(c) $f''(x)=\dfrac{2x(x^2-3)}{(x^2+1)^3}$, **so f is concave up on $(-\sqrt{3},0)$ and $(\sqrt{3},\infty)$; and concave down on $(-\infty,-\sqrt{3})$ and $(0,\sqrt{3})$. There are inflection points at $(-\sqrt{3},-\sqrt{3}/4)$, $(0,0)$, and $(\sqrt{3},\sqrt{3}/4)$.**

22. Sketch the graph of a function that satisfies all of the given conditions:

$f'(1) = f'(-1) = 0$,
$f'(x) < 0$ if $|x| < 1$,
$f'(x) > 0$ if $1 < |x| < 2$,
$f'(x) = 1$ if $|x| > 2$,
$f''(x) < 0$ if $-2 < x < 0$,
Inflection point (0,1)

***One possibility is shown to the right**



3.4

4. For the function g whose graph is given, state the following:

(a) $\lim_{x\to\infty} g(x) = \mathbf{2}$

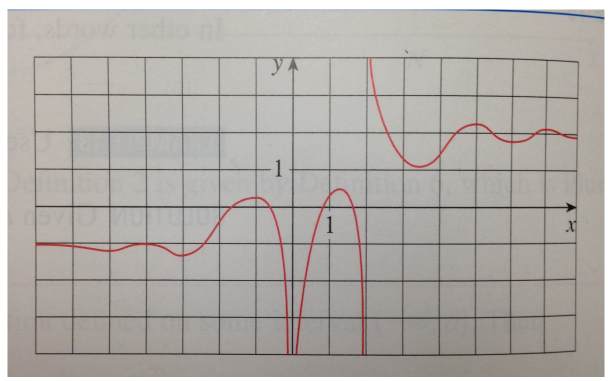
(b) $\lim_{x\to-\infty} g(x) = \mathbf{-1}$

(c) $\lim_{x\to 0} g(x) = -\infty$

(d) $\lim_{x\to 2^-} g(x) = -\infty$

(e) $\lim_{x\to 2^+} g(x) = \infty$

(f) The equations of the asymptotes $\boldsymbol{x = 0,\ x = 2}$, $\boldsymbol{y = -1,\ y = 2}$



12. Find the limit or show that it does not exist: $\lim_{x\to-\infty} \dfrac{4x^3+6x^2-2}{2x^3-4x+5}$

$$\lim_{x\to-\infty} \frac{4x^3+6x^2-2}{2x^3-4x+5} = \lim_{x\to-\infty} \frac{(4x^3+6x^2-2)/x^3}{(2x^3-4x+5)/x^3} = \lim_{x\to-\infty} \frac{4+6/x-2/x^3}{2-4/x^2+5/x^3} = \frac{4+0-0}{2-0+0} = 2$$