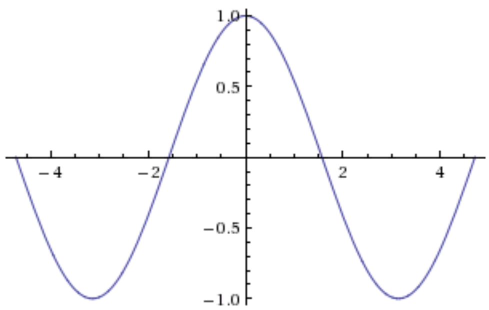
Week 7 Homework (Answers from Stewart's Solution Manuel)

3.1

22. Sketch the graph of f by hand and use your sketch to find the absolute and local maximum and minimum of f.

$f(x)$: $\cos(t)$

$-\frac{3\pi}{2} \leq t \leq \frac{3\pi}{2}$



Absolute *and* local max: $f(0) = 1$
Absolute *and* local min: $f(\pm\pi) = -1$

46. Find the absolute maximum and minimum of f on the given interval.

$f(x) = 5 + 54x - 2x^3$ on $[0,4]$

$f'(x) = 54 - 6x^2 = 6(3 + x)(3 - x)$. So $f(x) = 0$ if and only if $x = \pm 3$ (-3 not in interval)

$f(0) = 5, f(3) = 113$, and $f(4) = 93$

So $f(3) = 113$ is the absolute maximum, $f(0) = 5$ is the absolute minimum.

52. Find the absolute maximum and minimum of f on the given interval.

$f(x) = \frac{x}{x^2-x+1}$ on $[0,3]$

$f'(x) = \frac{(x^2-x+1)+x(2x-1)}{(x^2-x+1)^2} = \frac{(1+x)(1-x)}{(x^2-x+1)^2}$. So $f(x) = 0$ if and only if $x = \pm 1$ (-1 not in interval)

$f(0) = 0, f(1) = 1$, and $f(3) = \frac{3}{7}$

So $f(1) = 1$ is the absolute maximum, $f(0) = 0$ is the absolute minimum.

3.2

10. Verify that the function satisfies the hypothesis of the Mean Value Theorem on the interval. Then find all numbers c that satisfy the conclusion of the MVT.

$f(x) = x^3 - 3x + 2$ on $[-2,2]$

$f(x)$ is continuous on $[-2,2]$ and differentiable on$(-2,2)$ because polynomials always are!

MVT: $f'(c) = \frac{f(b)-f(a)}{b-a}$.

$f'(c) = 3c^2 - 3$ and $\frac{f(b)-f(a)}{b-a} = \frac{f(2)-f(-2)}{2-(-2)} = \frac{4-0}{4} = 1$

Thus, $3c^2 = 4$, **giving $\boldsymbol{c = \pm\frac{2}{\sqrt{3}}}$, both in the interval!**

18. Show that the equation has exactly one real root: $2x - 1 - \sin x = 0$.

Let $f(x) = 2x - 1 - \sin x$. Since $f(0) = -1 < 0$ and $f\left(\frac{\pi}{2}\right) = \pi - 2 > 0$, the **Intermediate Value Theorem** says that there is a number c in $\left(0, \frac{\pi}{2}\right)$ such that $f(c) = 0$. Thus, **the equation has at least one real root**.

More than one root? Suppose the equation has distinct real roots a and b with $a < b$. Then $f(a) = f(b) = 0$. Since f is continuous and differentiable on (a, b), Rolle's Theorem implies that there is a number r in (a, b) such that $f'(r) = 0$.
But $f'(r) = 2 - \cos r > 0$ since $\cos r \leq 1$. **This is contradiction let's us state that *f* has exactly one root**, not two**.**