# Solutions to Some Problems of Homework 5:

## Section 2.4:

12. Differentiate.

$$y = \frac{\cos x}{1 - \sin x} \qquad \text{(Use Quotient Rule)}$$

$$y' = \frac{(-\sin x)(1-\sin x) - (-\cos x).\cos x}{(1-\sin x)^2}$$

$$= \frac{-\sin x + \sin^2 x + \cos^2 x}{(1-\sin x)^2} = \frac{-\sin x + 1}{(1-\sin x)^2} = \frac{1}{1-\sin x}$$

44. Find the limit.

$$\lim_{x \to 0} \frac{\sin 3x \sin 5x}{x^2} \qquad \left(\text{Recall: } \lim_{x\to 0} \frac{\sin x}{x} = 1\right)$$

$$\lim_{x \to 0} \frac{\sin 3x \sin 5x}{x^2} = \lim_{x\to 0} \frac{\sin 3x}{x} \cdot \frac{\sin 5x}{x}$$

$$= \lim_{x\to 0} \frac{\sin 3x}{x} \cdot \frac{3}{3} \cdot \frac{\sin 5x}{x} \cdot \frac{5}{5}$$

$$= \lim_{x\to 0} \frac{\sin 3x}{3x} \cdot 3 \cdot \frac{\sin 5x}{5x} \cdot 5$$

$$= 15 \cdot \lim_{x\to 0} \frac{\sin 3x}{3x} \cdot \lim_{x\to 0} \frac{\sin 5x}{5x}$$

$$= 15$$

Section 2.5:

10. Find the derivative of the function.

$$f(x) = \frac{1}{(1+\sec x)^2}$$

$$f(x) = (1+\sec x)^{-2}$$

$$f'(x) \underset{\text{Use Chain Rule}}{=} (-2)\cdot(1+\sec x)^{-3}\cdot(\sec x \tan x)$$

$$= -\frac{2\sec x \tan x}{(1+\sec x)^3}$$

64. A table of values for $f$, $g$, $f'$ and $g'$ is given.

| $x$ | $f(x)$ | $g(x)$ | $f'(x)$ | $g'(x)$ |
|---|---|---|---|---|
| 1 | 3 | 2 | 4 | 6 |
| 2 | 1 | 8 | 5 | 7 |
| 3 | 7 | 2 | 7 | 9 |

a. If $F(x) = f(f(x))$, find $F'(2)$

b. If $G(x) = g(g(x))$, find $G'(3)$.

a. Let $F(x) = f(f(x))$

$$F'(x) \underset{\text{Chain rule}}{=} f'(f(x))\cdot f'(x)$$

$f(2)=1$, $f'(2)=5$, $f'(1)=4$

So $F'(2) = f'(f(2))\cdot f'(2) = 5\cdot f'(1) = 5\cdot 4 = 20$

b. $G(x) = g(g(x))$

$G'(x) = g'(g(x))\cdot g'(x)$

$g(3)=2$, $g'(3)=9$, $g'(2)=7$

So $G'(3) = g'(g(3))\cdot g'(3) = g'(2)\cdot 9 = 7\times 9 = 63$

Section 2.6:

12. Find $\frac{dy}{dx}$ by implicit Differentiation.

$$\cos(xy) = 1 + \sin y$$

Differentiate both sides with respect to x:

$$-\sin(xy) \cdot \frac{d}{dx}(xy) = \cos y \cdot \frac{dy}{dx}$$

$$-\sin(xy) \cdot \left[1.y + x\frac{dy}{dx}\right] = \cos y \frac{dy}{dx}$$

$$-y\sin(xy) - x\sin(xy) \cdot \frac{dy}{dx} = \cos y \cdot \frac{dy}{dx}$$

$$\frac{dy}{dx}\left[\cos y + x\sin(xy)\right] = -y\sin(xy)$$

and $$\frac{dy}{dx} = \frac{-y\sin(xy)}{\cos y + x\sin(xy)}$$