Solution for Selected Problems of HW 2:

Section 1.5:

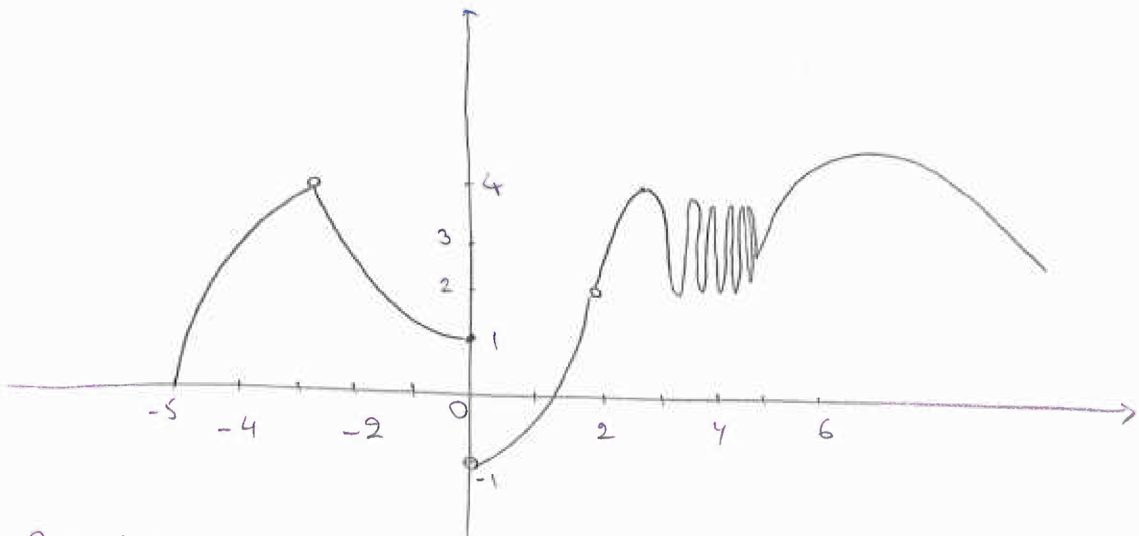
6- For the function h whose graph is given, state the value of each quantity, if it exists. If it does not exist, explain why.

a- $\lim_{x \to -3^-} h(x)$ b- $\lim_{x \to -3^+} h(x)$ c- $\lim_{x \to -3} h(x)$

d. $h(-3)$ e- $\lim_{x \to 0^-} h(x)$ f- $\lim_{x \to 0^+} h(x)$

g. $\lim_{x \to 0} h(x)$ h- $h(0)$ i- $\lim_{x \to 2} h(x)$

j- $h(2)$ k. $\lim_{x \to 5^+} h(x)$ l- $\lim_{x \to 5^-} h(x)$



a. $\lim_{x \to -3^-} h(x) = 4$

b. $\lim_{x \to -3^+} h(x) = 4$

c. $\lim_{x \to -3} h(x) = 4$

d- $h(-3)$ is not defined, so it doesn't exist.

e- $\lim_{x \to 0^-} h(x) = 1$

f- $\lim_{x \to 0^+} h(x) = -1$

g- $\lim_{x \to 0} h(x) =$ DNE because $\lim_{x \to 0^-} h(x) \neq \lim_{x \to 0^+} h(x)$

h- $h(0) = 1$

i- Since $\lim_{x \to 2^-} h(x) = \lim_{x \to 2^+} h(x) = 2$ then $\lim_{x \to 2} h(x) = 2$

j- $h(2)$ is not defined, so it doesn't exist.

k. $\lim_{x \to 5^+} h(x) = 3$

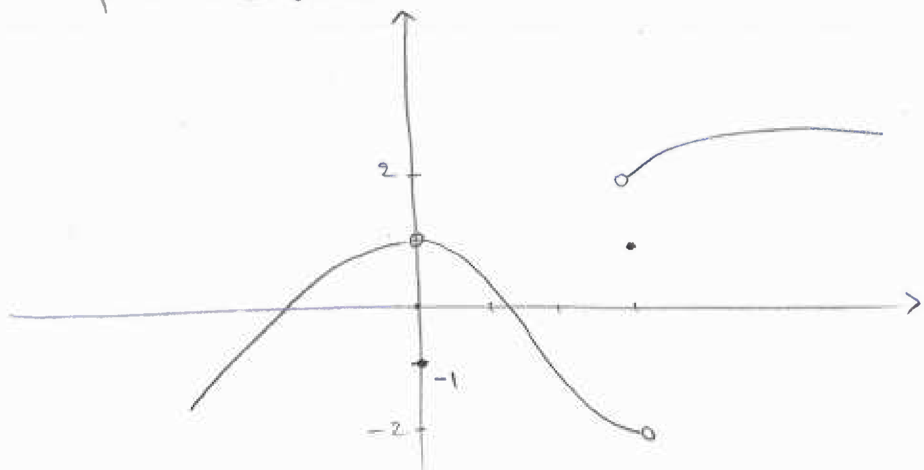
l- $\lim_{x \to 5^-} h(x) =$ DNE because $h(x)$ doesnot approach one number as x approaches 5 from the left

16- Sketch the graph of an example of a function f that satisfies all of the given conditions.

$\lim_{x \to 0} f(x) = 1$ $\quad \lim_{x \to 3^-} f(x) = -2$ $\quad \lim_{x \to 3^+} f(x) = 2$

$f(0) = -1$ $\quad f(3) = 1$

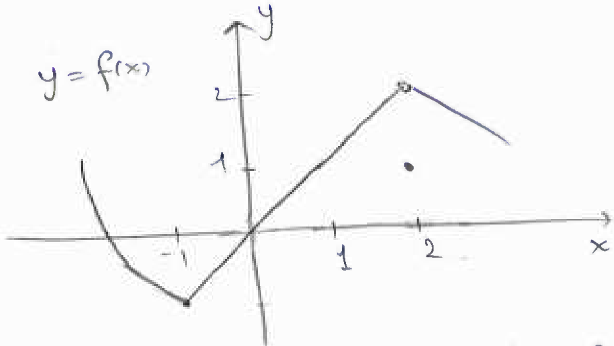
One acceptable answer:

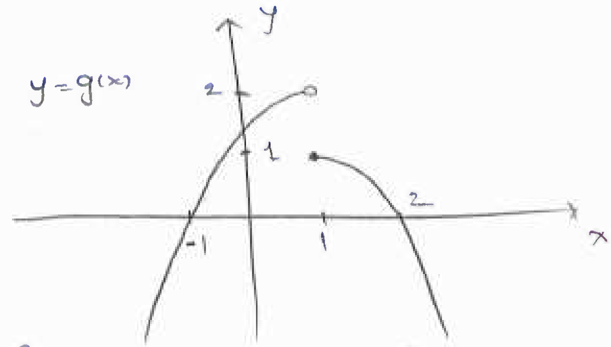


Note: Other answers would be considered true as long as your graph satisfies all above conditions.

Section 1.6:

2. The graphs of f and g are given. Use them to evaluate each limit, if it exists. If the limit does not exist, explain why.





a. $\lim_{x\to 2}[f(x)+g(x)] = \lim_{x\to 2} f(x) + \lim_{x\to 2} g(x) = 2+0=2$

b. $\lim_{x\to 1}[f(x)+g(x)]$

$\lim_{x\to 1} g(x) = \text{DNE}$ because $\lim_{x\to 1^-} g(x) \neq \lim_{x\to 1^+} g(x)$, so

$\lim_{x\to 1}[f(x)+g(x)]$ DNE.

c. $\lim_{x\to 0}[f(x)\,g(x)] = \left(\lim_{x\to 0} f(x)\right)\cdot\left(\lim_{x\to 0} g(x)\right) = (0)(1.3) = 0$

d. $\lim_{x\to -1} \dfrac{f(x)}{g(x)}$

Since $\lim_{x\to -1} g(x) = 0$ and g is in the denominator, but $\lim_{x\to -1} f(x) = -1 \neq 0$

So, $\lim_{x\to -1} \dfrac{f(x)}{g(x)} = \text{DNE}$

e. $\lim_{x\to 2}[x^3 f(x)] = \left[\lim_{x\to 2} x^3\right]\left[\lim_{x\to 2} f(x)\right] = 2^3 \cdot 2 = 16$

f. $\lim_{x\to 1}\sqrt{3+f(x)} = \sqrt{3+\lim_{x\to 1} f(x)} = \sqrt{3+1} = \sqrt{4} = 2$

8. Evaluate the limit and justify each step by indicating the appropriate Limit Law(s).

$$\lim_{t\to 2}\left(\frac{t^2-2}{t^3-3t+5}\right)^2 = \left(\lim_{t\to 2}\frac{t^2-2}{t^3-3t+5}\right)^2 \quad \text{[Limit Law 6]}$$

$$= \left(\frac{\lim_{t\to 2}(t^2-2)}{\lim_{t\to 2}(t^3-3t+5)}\right)^2 \quad \text{[Law 5]}$$

$$= \left(\frac{\lim_{t\to 2} t^2 - \lim_{t\to 2} 2}{\lim_{t\to 2} t^3 - 3\lim_{t\to 2} t + \lim_{t\to 2} 5}\right)^2 \quad \text{[Laws 1, 2 \& 3]}$$

$$= \left(\frac{4-2}{8-3(2)+5}\right)^2 \quad \text{[Laws 9, 7 \& 8]}$$

$$= \left(\frac{2}{7}\right)^2 = \frac{4}{29}$$

38. If $2x \le g(x) \le x^4 - x^2 + 2$ for all x, evaluate $\lim_{x\to 1} g(x)$.

We have:

(1) $\lim_{x\to 1} 2x = 2$

(2) $\lim_{x\to 1} x^4 - x^2 + 2 = 1^4 - 1^2 + 2 = 2$

Since $2x \le g(x) \le x^4 - x^2 + 2$ for all x, then

$$\lim_{x\to 1} 2x \le \lim_{x\to 1} g(x) \le \lim_{x\to 1}(x^4 - x^2 + 2)$$

and $\lim_{x\to 1} 2x = \lim_{x\to 1}(x^4 - x^2 + 2) = 2$, so

$\lim_{x\to 1} g(x) = 2$ by Squeeze theorem.