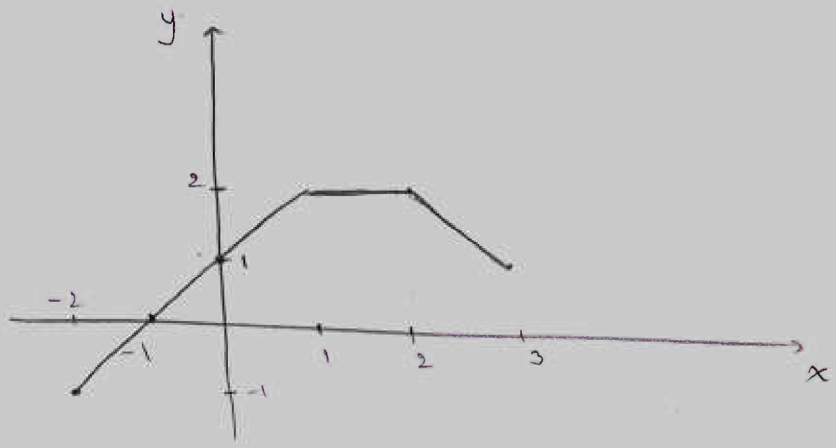
Solution for Selected Problems of HW 1:

Section 1.3:

4- The graph of f is given. Draw the graphs of the following functions.

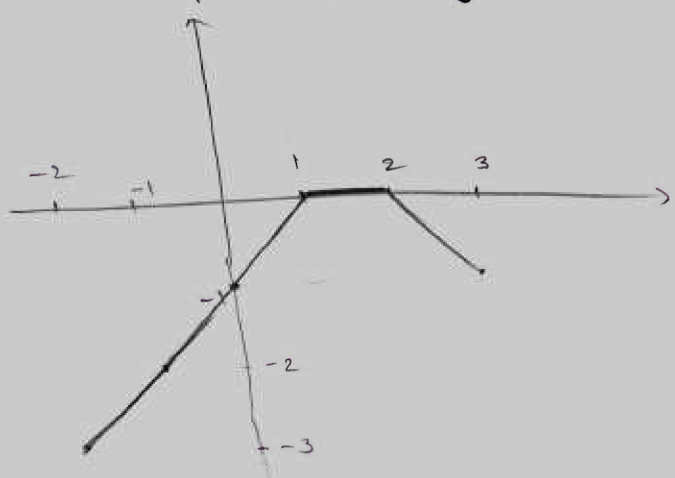
a- $y = f(x) - 2$ b- $y = f(x-2)$

c- $y = -2f(x)$ d- $y = f(\frac{1}{3}x) + 1$

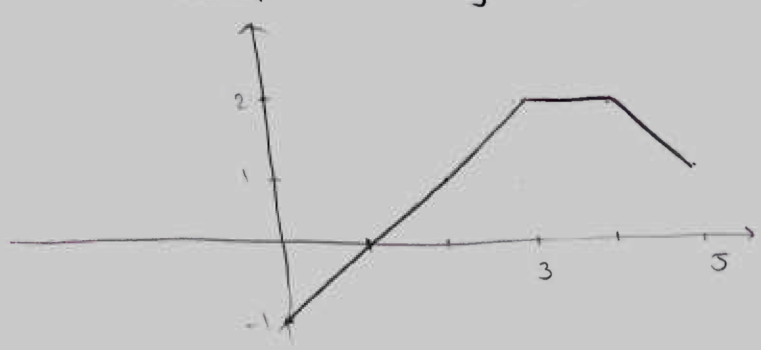


Answer:

a- $y = f(x) \underbrace{-\,2}_{\text{Shift downward by 2 units}}$



b- $y = f(\underbrace{x-2}_{\text{shift to the right by 2 units}})$

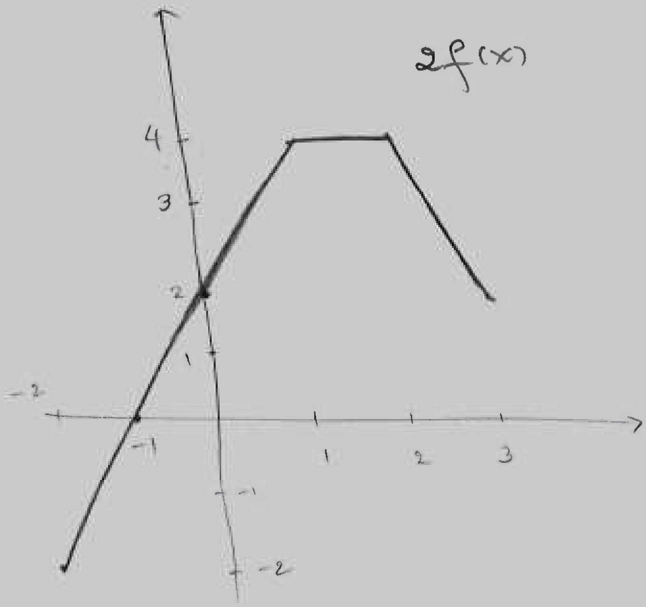


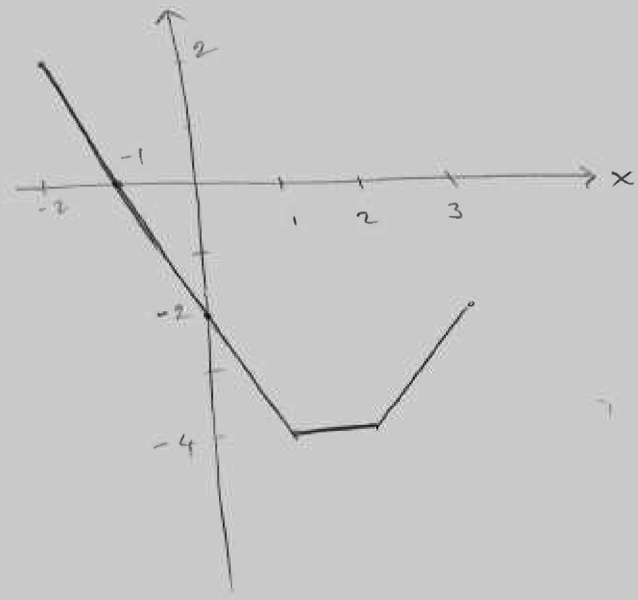
flip across x-axis

c. $y = -2f(x)$

Vertical stretch by a factor of 2

$-2f(x)$



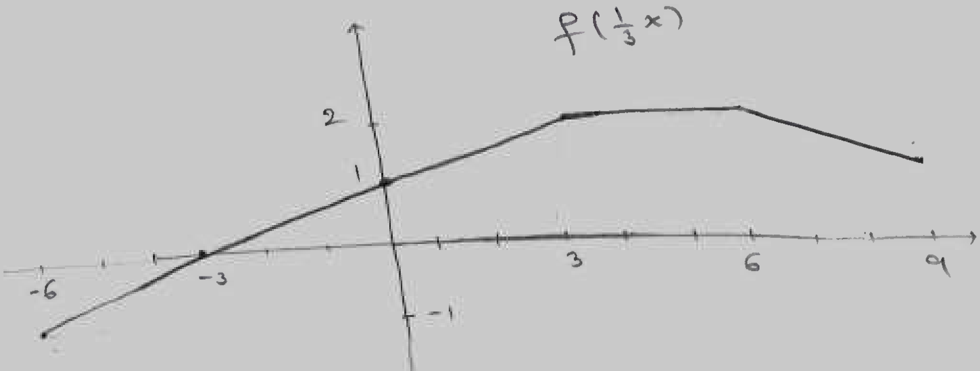


d. $y = f(\frac{1}{3}x) + 1$

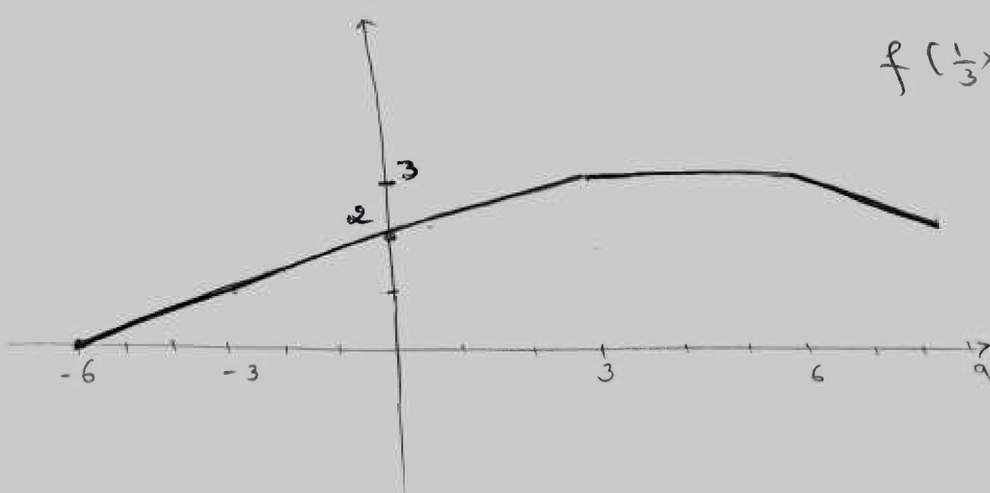
Horizontal stretch by 3 units

shift up by 1 unit

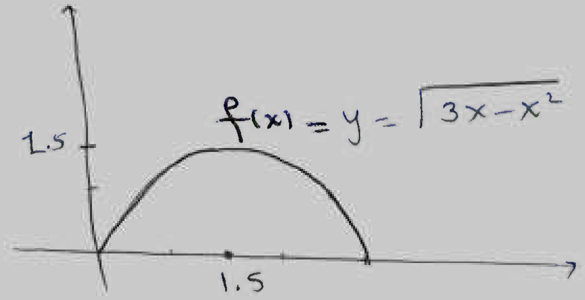
$f(\frac{1}{3}x)$



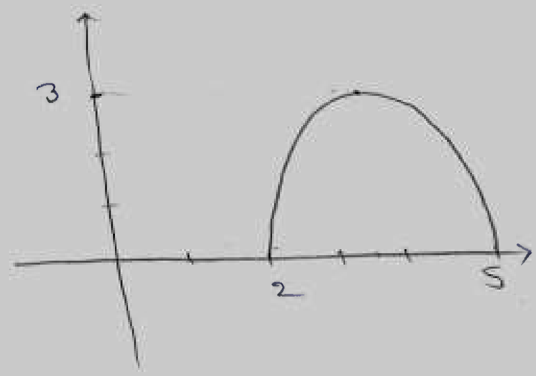
$f(\frac{1}{3}x) + 1$



6- The graph of $y = \sqrt{3x - x^2}$ is given.
Use transformations to create a function whose graph is as shown.



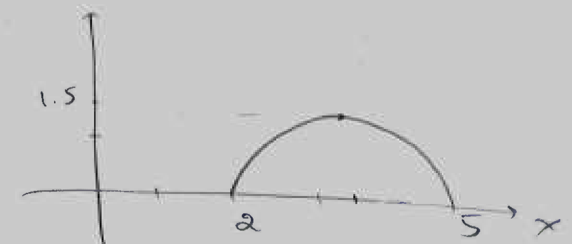
Given Graph.



Answer:

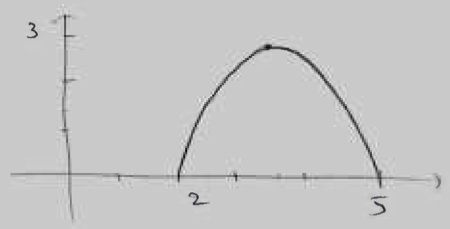
<u>Step 1</u>: Shift to the right by 2 units

$f(x) \longrightarrow f(x-2)$



<u>Step 2</u>. Vertical stretch by a factor of 2

$f(x-2) \longrightarrow 2 \cdot f(x-2)$



So, new function is

$$2 \cdot f(x-2) = 2 \cdot \sqrt{3(x-2) - (x-2)^2} = 2 \sqrt{-x^2 + 7x - 10}$$

$\uparrow$

$f(x) = \sqrt{3x - x^2}$

50. Use the table to evaluate each expression.

a. $f(g(1))$ b. $g(f(1))$ c. $f(f(1))$

d. $g(g(1))$ e. $(g \circ f)(3)$ f. $(f \circ g)(6)$

x	1	2	3	4	5	6
$f(x)$	3	1	4	2	2	5
$g(x)$	6	3	2	1	2	3

Answer:

a. $f(g(1)) = f(6) = 5$

b. $g(f(1)) = g(3) = 2$

c. $f(f(1)) = f(3) = 4$

d. $g(g(1)) = g(6) = 3$

e. $(g \circ f)(3) = g(f(3)) = g(4) = 1$

f. $(f \circ g)(6) = f(g(6)) = f(3) = 4.$

Section 1.4:

2. A cardiac monitor is used to measure the heart rate of a patient after surgery. It compiles the number of heart beats after t-minutes.

When the data in the table are graphed, the slope of the tangent line represents the heart rate in beats per minute.

t (min)	36	38	40	42	44
Heart beats	2530	2661	2806	2948	3080

The monitor estimates this value by calculating the slope of a secant line. Use the data to estimate the patient's heart rate after 42 minutes using the secant line between the points with the given values of t.

a. $t=36$ and $t=42$

b. $t=38$ and $t=42$

c. $t=40$ and $t=42$

d. $t=42$ and $t=44$

What are your conclusions?

Answer:

a. $$\text{slope} = \frac{2948-2530}{42-36} = \frac{418}{6} \approx 69.67$$

b. $$\text{slope} = \frac{2948-2661}{42-38} = \frac{287}{4} = 71.75$$

c. $$\text{slope} = \frac{2948-2806}{42-40} = \frac{142}{2} = 71$$

d. $$\text{slope} = \frac{3080-2948}{44-42} = \frac{132}{2} = 66$$

From the data, we see that the patient's heart rate is decreasing from 71 to 66 heartbeats/minute after 42 minutes. After being stable for a while, the patient's heart rate is dropping.

5- If a ball is thrown into the air with a velocity of 40 ft/s, its height in feet t seconds later is given by $y = 40t - 16t^2$.

a- Find the average velocity for the time period beginning when $t = 2$ and lasting:

i- 0.5 second ii- 0.1 second

iii- 0.05 second iv- 0.01 second

b- Estimate the instantaneous velocity when $t = 2$.

a- $y = y(t) = 40t - 16t^2$.

At $t = 2$, $y = 40(2) - 16(2)^2 = 16$.

The average velocity between times 2 and 2+h is

$$V_{ave.} = \frac{y(2+h) - y(2)}{(2+h) - 2} = \frac{[40(2+h) - 16(2+h)^2] - 16}{h}$$

$$= \frac{-24h - 16h^2}{h} = -24 - 16h \quad \text{if } h \neq 0.$$

i- $[2, 2.5]$: $h = 0.5$, $V_{ave.} = -32$ ft/s

ii- $[2, 2.1]$: $h = 0.1$, $V_{ave.} = -25.6$ ft/s

iii- $[2, 2.05]$: $h = 0.05$, $V_{ave.} = -24.8$ ft/s

iv- $[2, 2.01]$: $h = 0.01$, $V_{ave.} = -24.16$ ft/s

b- The instantaneous velocity when $t = 2$ (h approaches 0) is -24 ft/s.