<u>Week 14/15 Homework</u> (Answers from Stewart's Solution Manuel)

<u>5.1</u>

<u>18.</u> Sketch the region enclosed by the given curves and find its area.

$y = \sqrt{x-1}$, $x - y = 1$.

The curve is a square root function shifted to the right 1 unit intersected with a line of slope 1 shifted down 1 unit.

Finding the intersections: $x - y = 1 \quad \Rightarrow \quad y = x - 1$. These intersect when $\sqrt{x-1} = x - 1$.

$\sqrt{x-1} = x - 1 \quad \Rightarrow \quad x - 1 = (x-1)^2 = x^2 - 2x + 1 \quad \Rightarrow \quad x^2 - 3x + 2 = 0$. So $(x-2)(x-1) = 0$, meaning the curves intersect at 2 and 1.

Thus, $A = \int_1^2 \left[\sqrt{x-1} - (x-1)\right] dx = \left[\frac{2}{3}(x-1)^{3/2} - \frac{1}{2}(x-1)^2\right]_1^2 =$

$$\frac{2}{3}(1)^{3/2} - \frac{1}{2}(1)^2 - \frac{2}{3}(0)^{3/2} + \frac{1}{2}(0)^2 = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$$

<u>5.2</u>

<u>10.</u> Find the volume of the solid bounded by the curves rotated around the given line.

Bounded by $y = \frac{x^2}{4}$ and $x = 2$ and $y = 0$ rotated about the y-axis.

A cross-section is a washer with inner radius $x = 2\sqrt{y}$ and outer radius 2, so its area is $A(y) = \pi\left[(2)^2 - \left(2\sqrt{y}\right)^2\right] = 4\pi(1-y)$.

Thus, $V = \int_0^1 A(y)\,dy = 4\pi\int_0^1 (1-y)\,dy = 4\pi\left[y - \frac{y^2}{2}\right]_0^1 = 4\pi\left((1)^2 - \frac{(1)^2}{2}\right) = 4\pi\left(\frac{1}{2}\right) = 2\pi$

26. Find the volume of the solid bounded by the curves rotated around the given line.

Bounded by $y = \sqrt[4]{x}$ and $x = 0$ rotated about the line $y = 1$ (Uses in-book pictures).

$$V = \int_0^1 A(x)dx = \int_0^1 \pi\left(1 - \sqrt[4]{x}\right)^2 dx = \pi\int_0^1 \left(1 - 2x^{1/4} + x^{1/2}\right)dx = \pi\left(x - \frac{8}{5}x^{5/4} + \frac{2}{3}x^{3/2}\right)\Bigg|_0^1$$

$$= \pi\left(1 - \frac{8}{5} + \frac{2}{3}\right) = \frac{\pi}{15}$$

5.3

6. Use the method of shells to find the volume generated by the curve rotated about the y −axis.

Bounded by $y = 4x - x^2$ and $y = x$.

Finding intersection points: $4x - x^2 = x \quad \Rightarrow \quad 0 = x^2 - 3x = x(x-3)$. So the functions intersect at 0 and 3.

Thus, $V = \int_0^3 2\pi x\left[\left(4x - x^2\right) - x\right] dx = 2\pi\int_0^3 3x^2 - x^3 \, dx = 2\pi\left(x^3 - \frac{x^4}{4}\right)\Bigg|_0^3$

$$= 2\pi\left((3)^3 - \frac{(3)^4}{4}\right) - 2\pi\left((0)^3 - \frac{(0)^4}{4}\right) = 2\pi\left(27 - \frac{81}{4}\right) = \frac{27\pi}{2}$$

30. Describe the solid described by the function $2\pi\int_0^2 \frac{y}{1-y^2}\, dy$.

$2\pi\int_0^2 \frac{y}{1-y^2}\, dy = \int_0^2 2\pi y\left(\frac{1}{1-y^2}\right) dy$, so **the solid is obtained by rotating the region** $0 \le x \le \frac{1}{1-y^2}$ **from** $0 \le y \le 2$ **about the** $\boldsymbol{x}$ **−axis using cylindrical shells.**