<u>Week 10 Homework</u> (Answers from Stewart's Solution Manuel)

<u>4.4</u>

<u>26.</u> Evaluate the integral $\int_1^2 \left(\frac{1}{x^2} - \frac{4}{x^3} \right) dx$.

$$\int_1^2 \left(\frac{1}{x^2} - \frac{4}{x^3} \right) dx = \int_1^2 \left(x^{-2} - 4x^{-3} \right) dx = \left(-x^{-1} + 2x^{-2} \right) \Big|_1^2 = \left(-\frac{1}{x} + \frac{2}{x^2} \right) \Big|_1^2 = \left(-\frac{1}{2} + \frac{1}{2} \right) - \left(-1 + 2 \right) = -1$$

<u>36.</u> Evaluate the integral $\int_1^8 \frac{x-1}{\sqrt[3]{x^2}} \, dx$.

$$\int_1^8 \frac{x-1}{\sqrt[3]{x^2}} \, dx = \int_1^8 x^{1/3} - x^{-2/3} \, dx = \left(\frac{x^{4/3}}{4/3} - \frac{x^{1/3}}{1/3} \right) \Big|_1^8 = \left(\frac{3x^{4/3}}{4} - 3x^{1/3} \right) \Big|_1^8 = \left(\frac{3}{4}(16) - 3(2) \right) - \left(\frac{3}{4} - 3 \right) = \frac{33}{4}$$

<u>4.5</u>

<u>18.</u> Evaluate the indefinite integral $\int \cos^4(\theta) \sin(\theta) d\theta$.

Let $u = \cos\theta$, so $du = -\sin\theta d\theta$. The integral becomes $-\int u^4 du = \frac{-u^5}{5} + C$. Subbing back in gives the final answer $\frac{-\cos^5\theta}{5} + C$.

<u>46.</u> Evaluate the definite integral $\int_{-\pi/3}^{\pi/3} x^4 \sin x \, dx$.

Since this is an *odd* function ($\sin x$) multiplied by an *even* function (x^4), making it ***odd***, over a *symmetric* interval, $\int_{-\pi/3}^{\pi/3} x^4 \sin x \, dx = 0$

<u>5.1</u>

5.1

22. Sketch the region enclosed by the given curves and find its area.

Curves: $y = x^3$, $y = x$.

Curves intersect when $x^3 = x$. Thus, $x^3 - x = 0 \quad \Rightarrow \quad x(x^2-1) = 0 \quad \Rightarrow \quad x(x-1)(x+1) = 0$.
The curves intersect at $x = 0,\ x = \pm 1$.

So, $A = 2\int_0^1 (x - x^3)\,dx$ (by symmetry) $= 2\left(\frac{x^2}{2} - \frac{x^4}{4}\right)\Big|_0^1 = 2\left(1/2 - 1/4\right) = \frac{1}{2}$