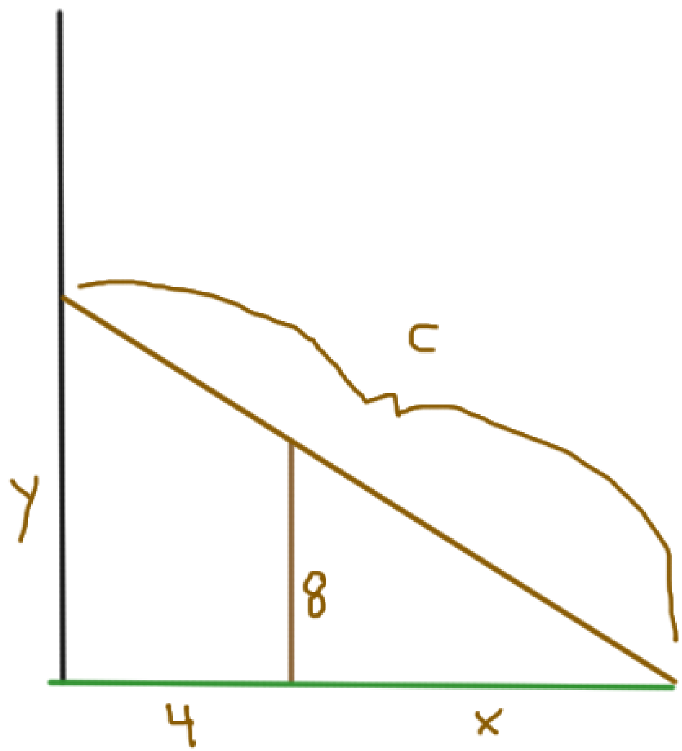
1. Exercise 3.7.38. A fence 8 ft tall runs parallel to a tall building at a distance of 4 ft from the building. What is the length of the shortest ladder that will reach from the ground over the fence to the wall of the building?



In the diagram, the building side is in black. We want to minimize c, the ladder length. The ladder forms a right triangle with the building side and the ground, so we can write $y^2 + (4+x)^2 = c^2$ by the Pythagorean Theorem. We use similar triangles to get rid of one of the variables x or y.

$$\frac{8}{x} = \frac{y}{4+x} \Rightarrow 32 + 8x = xy \Rightarrow y = \frac{32+8x}{x}.$$

Substitute this into the first equation:

$$\left(\frac{1024}{x^2} + \frac{512}{x} + 64\right) + \left(16 + 8x + x^2\right) = c^2$$

$$\Rightarrow \frac{1024 + 512x + 80x^2 + 8x^3 + x^4}{x^2} = c^2.$$

Let $f = c^2$.

$$f'(x) = \frac{(x^2)(512 + 160x + 24x^2 + 4x^3) - (1024 + 512x + 80x^2 + 8x^3 + x^4)(2x)}{x^4}$$

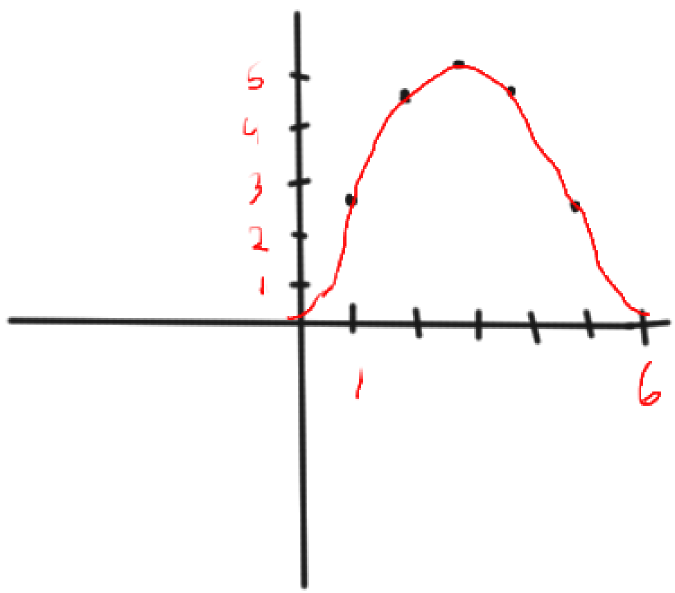
$$\Rightarrow f'(x) = \frac{2(x+4)(x^3 - 256)}{x^3}.$$

The only positive critical point is $4\sqrt[3]{4}$.

Then, $c = \sqrt{\frac{1024+512(4\sqrt[3]{4})+80(4\sqrt[3]{4})^2+8(4\sqrt[3]{4})^3+(4\sqrt[3]{4})^4}{(4\sqrt[3]{4})^2}}$.

2. Exercise 4.3.4

   (a) $g(0) = g(6) = 0$.
   (b) $g(1) \approx 2.75$. $g(2) \approx 4.5$. $g(3) \approx 5$. $g(4) \approx 4.5$. $g(5) \approx 2.75$.
   (c) $(0, 3)$.
   (d) $x = 3$.



   (e)
   (f) This should look like the graph of f.

3. Exercise 4.3.24

$$\int_1^8 x^{-2/3} dx = 3x^{1/3}|_1^8 = 3(2) - 3(1) = 3.$$

4. Exercise 4.4.6 Find the general indefinite integral.

$$\int (\sqrt{x^3} + \sqrt[3]{x^2}) dx = \frac{2}{5}x^{5/2} + \frac{3}{5}x^{5/3} + C.$$

5. Exercise 4.4.12 Find the general indefinite integral.

$$\int (u^2 + 1 + \frac{1}{u^2}) du = \frac{1}{3}u^3 + u - \frac{1}{u} + C.$$