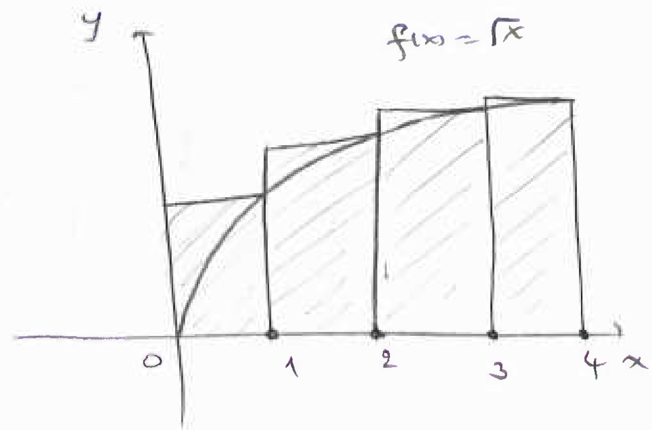
Solutions to some Selected Problems of HW 11:

Section 4.1:

4- a. Estimate the area under the graph of $f(x)=\sqrt{x}$ from $x=0$ to $x=4$ using four approximating rectangles and right endpoints. Sketch the graph and rectangles. Is your estimate an underestimate or an overestimate?

b. Repeat part (a) using left endpoints.

a. $\Delta x = \dfrac{4-0}{4} = 1$



$$R_4 = \sum_{i=1}^{4} f(x_i)\cdot \Delta x$$

$$= f(1)\cdot\Delta x + f(2)\cdot\Delta x + f(3)\,\Delta x + f(4)\,\Delta x$$

$$= \sqrt{1}+\sqrt{2}+\sqrt{3}+\sqrt{4} \approx 6.1463$$

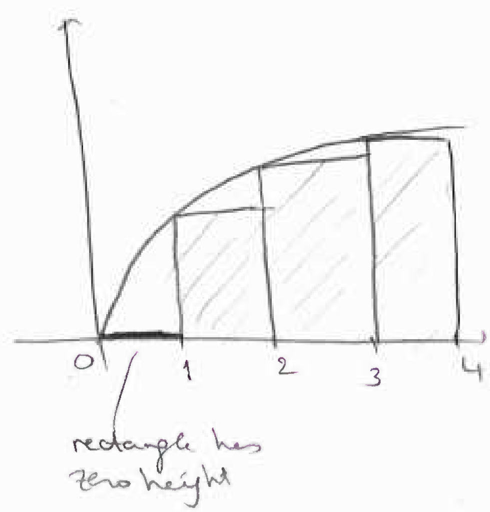
Since f is increasing on $[0,4]$, R_4 is an overestimate.

b. $$L_4 = \sum_{i=1}^{4} f(x_{i-1})\,\Delta x$$

$$= f(0)\cdot\Delta x + f(1)\,\Delta x + f(2)\,\Delta x + f(3)\,\Delta x$$

$$= 0+\sqrt{1}+\sqrt{2}+\sqrt{3}$$

$$\approx 4.1463$$



Since f is increasing on $[0,4]$, L_4 is an underestimate.

14. Speedometer readings for a motorcycle at 12-second intervals are given in the table.

a. Estimate the distance traveled by the motorcycle during this time period using the velocities at the beginning of the time intervals

b. Give another estimate using the velocities at the end of the time periods.

c. Are your estimates in parts (a) & (b) upper or lower estimates? Explain.

t (s)	0	12	24	36	48	60
v (ft/s)	30	28	25	22	24	27

$\Delta t = 12$ [Difference between 2 consequative times giving the width of every estimating rectangle]

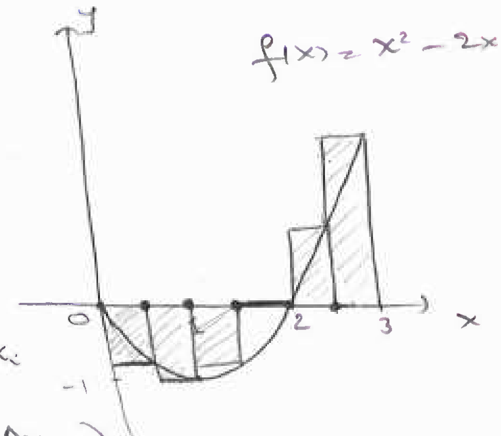
a. $d \approx L_5 = (30)(12) + (28)(12) + (25)(12) + (22)(12) + (24)(12)$
$= 1584$ ft

b. $d \approx R_5 = (28 + 25 + 22 + 24 + 27)12 = 1512$ ft.

c. The estimates are <u>neither</u> lower nor upper estimates since v is <u>neither</u> an increasing nor a decreasing function of t.

Section 4.2:

2- If $f(x) = x^2 - 2x$, $0 \le x \le 3$, evaluate the Riemann sum with $n=6$, taking the sample points to be right endpoints. What does the Riemann sum represent? Illustrate with a diagram.



$f(x) = x^2 - 2x$

$= x^2 - 2x + 1 - 1$

$= (x-1)^2 - 1$

$\Delta x = \frac{3-0}{6} = \frac{1}{2}$

Since we are using right endpoints, $x_i^* = x_i$

$R_6 = \sum_{i=1}^{6} f(x_i).\Delta x$ $\left(x_i = 0 + i.\Delta x = i.\frac{1}{2} = \frac{i}{2} \right)$

$= \Delta x [f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5) + f(x_6)]$

$= \Delta x [f(\frac{1}{2}) + f(1) + f(\frac{3}{2}) + f(2) + f(\frac{5}{2}) + f(3)]$

$= \frac{1}{2} [-\frac{3}{4} - 1 - \frac{3}{4} + 0 + \frac{5}{4} + 3] = \frac{1}{2} \cdot (\frac{7}{4}) = \frac{7}{8}$

The Riemann sum represents the sum of the areas of the two rectangles above the x-axis minus the sum of the areas of three rectangles below the x-axis, that is, the net area of the rectangles with respect to the x-axis.

30- Express the integral as a limit of Riemann sums. Do not evaluate the limit.

$$\int_0^{2\pi} x^2 \sin x \, dx$$

$f(x) = x^2 \sin x$

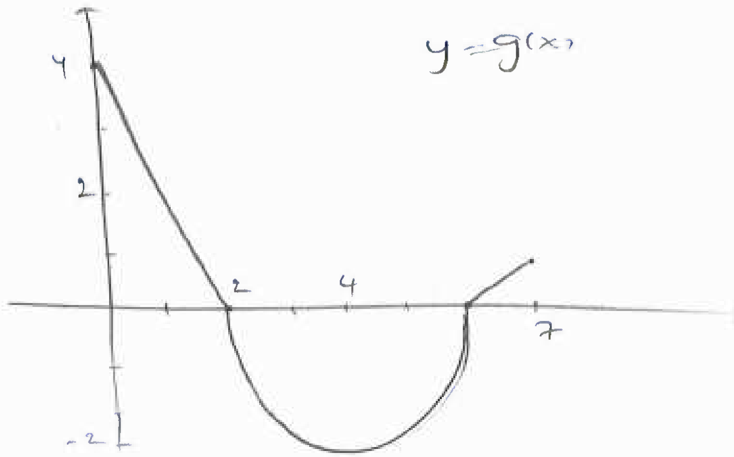
$a = 0$ $b = 2\pi$

$$\boxed{\begin{aligned} \Delta x &= \frac{2\pi - 0}{n} = \frac{2\pi}{n} \\ \& \quad x_i &= 0 + i\Delta x = \frac{2\pi i}{n} \end{aligned}}$$

So, $\int_0^{2\pi} x^2 \sin x \, dx = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \sum_{i=1}^{n} \left[\underbrace{\left(\frac{2\pi i}{n}\right)}_{x_i}^2 \cdot \sin\left(\frac{2\pi i}{n}\right) \right] \cdot \underbrace{\frac{2\pi}{n}}_{\Delta x}$

34. The graph of g consists of two straight lines and a semi-circle. Use it to evaluate each integral.

a. $\int_0^2 g(x)\,dx$ b. $\int_2^6 g(x)\,dx$ c. $\int_0^7 g(x)\,dx$



a. $\int_0^2 g(x)\,dx = \frac{1}{2}\cdot 4\cdot 2 = 4$ [area of triangle].

b. Area of semicircle of radius 2 is $\frac{1}{2}\pi 4 = 2\pi$

So, $\int_2^6 g(x)\,dx = -2\pi$ [negative of area of semicircle since area is below x-axis

c. $\int_6^7 g(x)\,dx = \frac{1}{2}\cdot 1\cdot 1 = \frac{1}{2}$ (area of triangle)

So, $\int_0^7 g(x)\,dx = \int_0^2 g(x)\,dx + \int_2^6 g(x)\,dx + \int_6^7 g(x)\,dx$

$= 4 - 2\pi + \frac{1}{2}$

$= \frac{9}{2} - 2\pi = 4.5 - 2\pi$