<u>Week 10 Homework</u> (Answers from Stewart's Solution Manuel)

<u>2.5</u>

28. Find the derivative of the function $f(x)=\dfrac{\cos(\pi x)}{\sin(\pi x)+\cos(\pi x)}$.

$$f(x)=\frac{\cos(\pi x)}{\sin(\pi x)+\cos(\pi x)} \quad \Rightarrow$$

$$f'(x)=\frac{\left(\sin(\pi x)+\cos(\pi x)\right)(-\pi\sin(\pi x))-\cos(\pi x)\left(\pi\cos(\pi x)-\pi\sin(\pi x)\right)}{\left(\sin(\pi x)+\cos(\pi x)\right)^2}$$

$$=\frac{-\pi\left(\sin^2(\pi x)+\cos^2(\pi x)\right)}{\left(\sin(\pi x)+\cos(\pi x)\right)^2} \quad = \quad \frac{-\pi}{\left(\sin(\pi x)+\cos(\pi x)\right)^2}$$

<u>OR</u> $= \dfrac{-\pi}{1+2\sin(\pi x)\cos(\pi x)}$

<u>2.8</u>

14. At noon, Ship A is 150 km away west of Ship B. Ship A is sailing East at 35 km/h and Ship B is sailing North at 25 km/h. How fast is the distance between the ships changing at 4:00 pm?

If z is the distance between the ships, we need to find dz/dt when $t = 4h$.

$z^2=(150-x)^2+y^2$. So, at 4 pm, $x=4(35)=140$ and $y=4(25)$, making $z=\sqrt{(150-140)^2+100^2}=\sqrt{10{,}100}=10\sqrt{101}$.

Moreover, $z^2=(150-x)^2+y^2 \quad \Rightarrow \quad 2z\dfrac{dz}{dt}=-2(150-x)\dfrac{dx}{dt}+2y\dfrac{dy}{dt}$.

So $\dfrac{dz}{dt}=\dfrac{1}{z}\left[(x-150)\dfrac{dx}{dt}+y\dfrac{dy}{dt}\right]=\dfrac{-10(35)+100(25)}{10\sqrt{101}}=\dfrac{215}{\sqrt{101}}$.

Thus, $\dfrac{dz}{dt}\approx 21.4$ **km/h.**

<u>3.7</u>

16. A rectangular storage container with an open top is to have a volume of 10 m^3. The length of its base is twice the width. Material for the base costs \$10 per square meter. Material for the sides costs \$6 per square meter. Find the cost of materials for the cheapest such container.

First, $V = lwh$, so $10 = (2w)(w)h = 2w^2h \quad \Rightarrow \quad h = 5/w^2$.

The cost is $C(w) = 10(2w^2) + 6[2(2wh) + 2(hw)] = 20w^2 + 36wh$. Plugging in our height… $C(w) = 20w^2 + 36w(5/w^2) = 20w^2 + 180/w$. We now just have to minimize this.

$C'(w) = 40w - 180/w^2 = 0 \quad \Rightarrow \quad w = \sqrt[3]{\frac{9}{2}}$ is the critical value (one should check this is a minimum), meaning $C\left(\sqrt[3]{\frac{9}{2}}\right) = 20\left(\sqrt[3]{\frac{9}{2}}\right)^2 + 180\Big/\left(\sqrt[3]{\frac{9}{2}}\right) \approx \$163.54$ **is the cheapest cost.**

34. A poster is to have an area of 180 in^2 with 1-inch margins at the bottom and sides and a 2-inch margin at the top. What dimensions will give the largest printed area.

First, $xy = 180$, so $y = 180/x$. The printed area is $A(x) = (x-2)(y-3)$
$= (x-2)(180/x - 3) = 186 - 3x - 360/x$. We simply have to maximize this…

$A'(x) = -3 + 360/x^2 = 0 \quad \Rightarrow \quad x^2 = 120 \quad \Rightarrow \quad x = 2\sqrt{30}$ (one should check this is a maximum).

And, $y = 180\Big/\left(2\sqrt{30}\right) = 90/\sqrt{30}$. **The dimensions are** $2\sqrt{30}$ **in. and** $90/\sqrt{30}$ **in.**

<u>3.9</u>

34. Find f from $f''(x) = 8x^3 + 5$ given that $f(1) = 0$ and $f'(1) = 8$.

$f''(x) = 8x^3 + 5 \quad \Rightarrow \quad f'(x) = 8\left(\frac{x^4}{4}\right) + 5x + C$. So, $f'(x) = 2x^4 + 5x + C$. Given that $f'(1) = 8$, we can solve for C… $f'(1) = 2(1)^4 + 5(1) + C = 8 \quad \Rightarrow \quad C = 1$. Thus, $f'(x) = 2x^4 + 5x + 1$.

Now for f. $f'(x) = 2x^4 + 5x + 1 \quad \Rightarrow \quad f(x) = \frac{2}{5}x^5 + \frac{5}{2}x^2 + x + D$. Given that $f(1) = 0$, we can solve for D… $f(1) = \frac{2}{5}(1)^5 + \frac{5}{2}(1)^2 + (1) + D = 0 \quad \Rightarrow \quad D = -1 - \frac{5}{2} - \frac{2}{5} = -\frac{39}{10}$.

Thus, $f(x) = \frac{2}{5}x^5 + \frac{5}{2}x^2 + x - \frac{39}{10}$ **.**