1(a) Find the absolute maximum and minimum values of the function $f(x) = (x^2 - 1)^3$ on the interval $[-1, 2]$.

Closed interval method.

$f'(x) = 3(x^2-1)^2(2x) = 0$

$\underline{x=0}$ or $x^2-1=0$

$\underline{x=\pm 1}$

critical points

evaluate: $f(-1) = 0$

$f(0) = -1$

$f(1) = 0$

$f(2) = 27$

$\boxed{\text{abs. min. value is } -1 \quad \text{abs. max. value is } 27}$

1(b) Explain why the function $g(x) = \sin x + x^3 + 2x$ has no local maxima or minima.

$g'(x) = \cos x + 3x^2 + 2$

$\geq -1 \qquad \geq 0 \qquad 2$

$g'(x)$ is always at least 1, so never zero. hence no critical points, so no local maxima or minima.

2(a) Consider the function $f(x) = \frac{x}{x+2}$.

- Verify that f satisfies the requirements for the Mean Value Theorem on the interval $[1, 4]$.
- What exactly does the Mean Value Theorem say about this situation? Find all numbers c that satisfy the conclusion of the theorem.

Since f is rational, it is continuous and differentiable on its domain ($\{x \in \mathbb{R} \mid x \neq -2\}$). So f is cont. on $[1,4]$ and diff. on $(1,4)$.

MVT says: there is a number c between 1 and 4 such that

$$f'(c) = \frac{f(4) - f(1)}{4-1}$$

finding c:

$$f'(x) = \frac{x+2-x}{(x+2)^2} = \frac{2}{(x+2)^2}.$$

$$\frac{f(4)-f(1)}{4-1} = \frac{\frac{2}{3} - \frac{1}{3}}{3} = \frac{1}{9}.$$

$$\frac{2}{(x+2)^2} = \frac{1}{9} \Rightarrow 18 = (x+2)^2$$

$$\boxed{\sqrt{18} - 2} = x$$

this is c

2(b) If $g(2) = 4$ and $2 \leq g'(x) \leq 6$ for all numbers x between 2 and 8, how big can $g(8)$ be? Explain.

MVT: $\frac{g(8)-g(2)}{8-2} \leq g'(c)$ some c in $(2, 8)$

≤ 6 since $g'(x) \leq 6$

So $g(8) - g(2) \leq 36$

$g(8) \leq 36 + g(2) = 40$

$\boxed{g(8) \text{ is at most } 40}$

3. The function $f(x) = x^4 + 4x^3$ has domain $\mathbb{R}$ and has no asymptotes. It has no symmetry and is not periodic.

Draw an accurate sketch of the graph of f, by computing carefully the following items: intercepts; intervals of increase/decrease; critical points and local maxima/minima; intervals of upward/downward concavity; inflection points.

Label all interesting points on the graph.

Intercepts: $f(0) = \underline{0} \leftarrow$ y-intercept

$0 = x^4 + 4x^3 = x^3(x+4) \Rightarrow \underline{x = 0, -4} \leftarrow$ x-intercepts

$f'(x) = 4x^3 + 12x^2 = 4x^2(x+3)$

C.P.s: $4x^2(x+3) = 0 \Rightarrow x = 0, -3$

f': $-$ | $+$ | $+$

f decr. -3 f incr. 0 f incr.

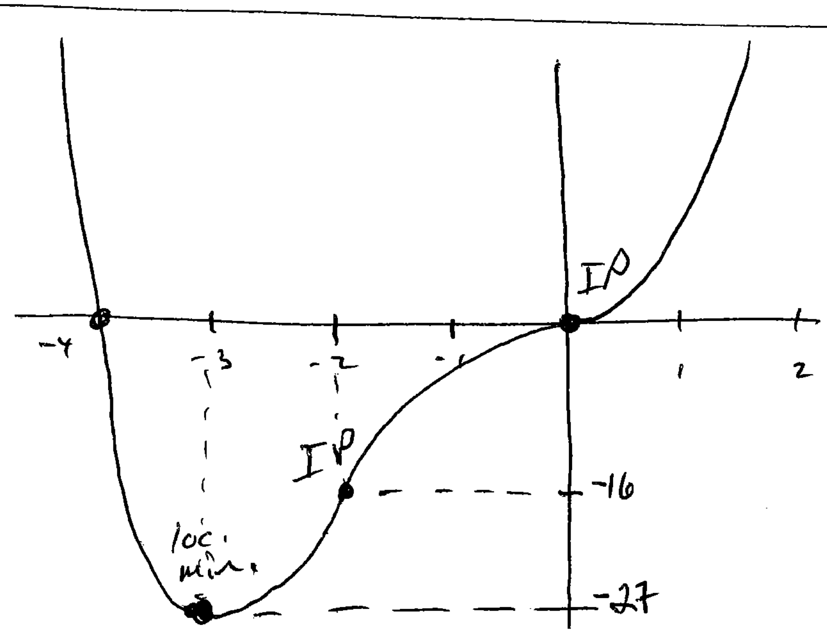
$f''(x) = 12x^2 + 24x = 12x(x+2)$

$12x(x+2) = 0 \Rightarrow x = 0, -2$

f'': $+$ | $-$ | $+$

f concave $\uparrow$ -2 f concave $\downarrow$ 0 f concave $\uparrow$

graph



$f(-2) = -16$

$f(-3) = -27$

4. Find all asymptotes (verical, horizontal, slant) for the following curves, and give brief explanations.

(a) $y = \dfrac{2x^2}{x^2+1}$ domain = $\mathbb{R}$, so no vertical asymptotes.

$$\lim_{x\to\infty} \frac{2x^2}{x^2+1} = \lim_{x\to\infty} \frac{2}{1+\frac{1}{x^2}} = 2$$

$$\lim_{x\to-\infty} \frac{2x^2}{x^2+1} = \lim_{x\to-\infty} \frac{2}{1+\frac{1}{x^2}} = 2.$$

$\boxed{\text{horizontal asymptote} \quad y=2}$

(b) $y = 2x - 3 + \dfrac{x}{x^2+1}$

$$\lim_{x\to\infty} \frac{x}{x^2+1} = \lim_{x\to\infty} \frac{\frac{1}{x}}{1+\frac{1}{x^2}} = \frac{0}{1}$$

So $\boxed{y = 2x-3 \text{ is a slant asymptote}}$

no vert. asymptote (domain is $\mathbb{R}$)

or horiz asymptote

$\left(\lim_{x\to\pm\infty} f(x) = \pm\infty\right)$

Extra Credit The graph of $f(x) = \frac{x^4+1}{x^2}$ has a *curved* asymptote. What is it? Can you draw the graph of f together with its asymptote?

$f(x) = x^2 + \frac{1}{x^2}$

and $\lim_{x\to\pm\infty} \frac{1}{x^2} = 0 \Rightarrow \boxed{y = x^2 \text{ is an asymptote}}.$