1. Consider the function $f(x) = x^2 - 5x$.

(a) Write the definition of $f'(3)$ for this function.

(b) Compute $f'(3)$ from the definition, showing all your work.

(c) Find an equation of the tangent line to the curve $y = f(x)$ at the point $(3, -6)$.

(a) $$\boxed{f'(3) = \lim_{h \to 0} \frac{[(3+h)^2 - 5(3+h)] - [3^2 - 5 \cdot 3]}{h}}$$

or $$\boxed{\lim_{x \to 3} \frac{(x^2 - 5x) - (3^2 - 5 \cdot 3)}{x - 3}}$$

(b)
$$f'(3) = \lim_{h \to 0} \frac{\cancel{9} + 6h + h^2 - \cancel{15} - 5h - \cancel{9} + \cancel{15}}{h}$$

$$= \lim_{h \to 0} \frac{h + h^2}{h} = \lim_{h \to 0} (1 + h) = 1.$$

So $\boxed{f'(3) = 1}$

(c) Slope $= 1$, point $= (3, -6)$

$$\boxed{y + 6 = 1(x - 3)}$$

2. Let $f(x) = \begin{cases} 4x-2 & \text{if } x \geq 2 \\ 2x+a & \text{if } x < 2. \end{cases}$

(a) Find $\lim_{x\to 2^-} f(x)$ and $\lim_{x\to 2^+} f(x)$, explaining your reasoning.

(b) What must be true for $f(x)$ to be continuous at $x=2$? Which, if any, values of a make $f(x)$ continuous at $x=2$?

(a) $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-}(2x+a)$ since $x < 2$ in this limit

$= 2(2)+a$ (Direct Subs. Property)

$= \boxed{4+a}$.

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}(4x-2)$ since $x > 2$ in this limit

$= 4(2)-2$ (Direct. Subs. Prop.)

$= \boxed{6}$.

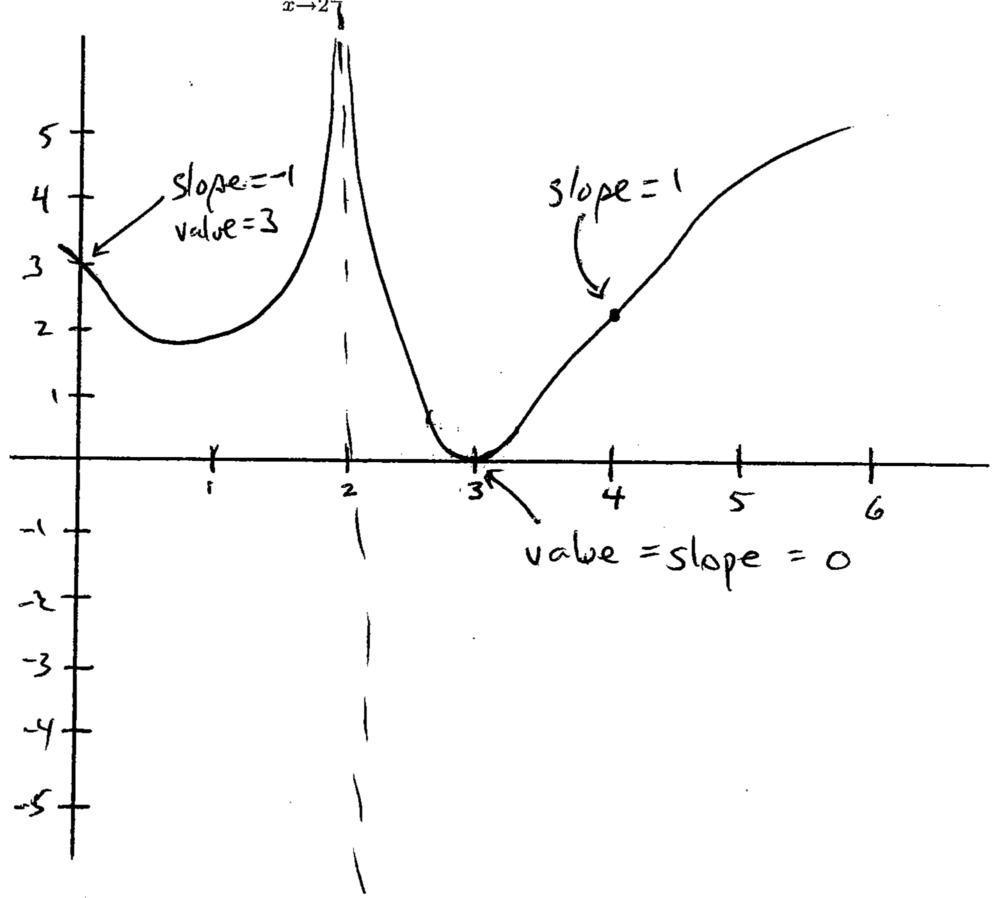
(b) It must be true that $\lim_{x\to 2} f(x)$ exists, and is equal to $f(2)$, which is $4(2)-2 = 6$.

For the limit to exist we must have $4+a = 6$, so $a=2$.

Then $\lim_{x\to 2} f(x) = 6 = f(2)$, so

$\boxed{a=2 \text{ makes } f \text{ continuous at } 2.}$

3(a) On the coordinate system below, sketch the graph of a function $f(x)$ satisfying the following conditions: $f(0) = 3$, $f'(0) = -1$, f is discontinuous at $x = 2$ but is continuous at all other $x \leq 5$, $f(3) = 0$, $f'(3) = 0$, $f'(4) = 1$, $\lim_{x \to 2} f(x) = \infty$.



(b) Suppose the function $g(x)$ is continuous on $[5, 10]$ and $g(5) = 4$, $g(10) = 6$. What, exactly, does the Intermediate Value Theorem say about $g(x)$ in this situation?

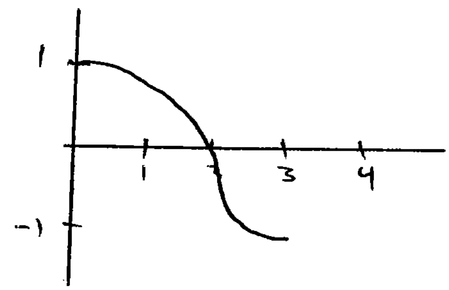
It says that for every number N between 4 and 6, there is a number c between 5 and 10 such that g(c) = N.

4. Consider the function $f(x)$ whose graph is shown here.

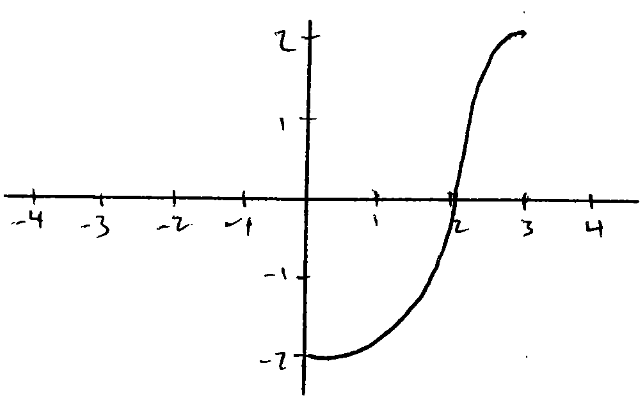
(a) Graph the function $-2f(x)$.

(b) Graph the function $-2f(x+3)$.

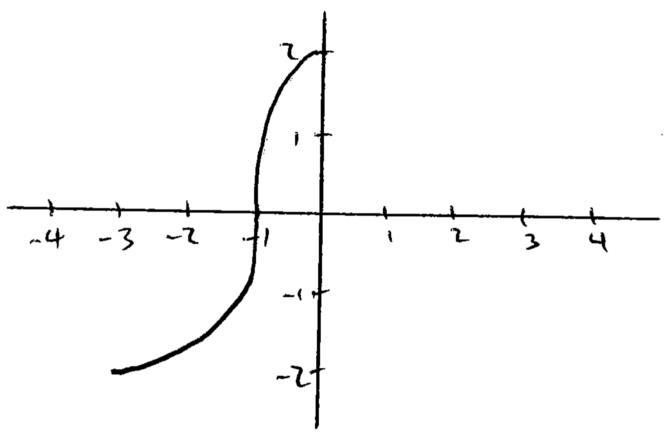
(c) Suppose $g(x)$ is an even function which agrees with $f(x)$ on $[0,3]$. Draw the graph of $g(x)$.



(a)



(b)



(c)