1. Consider the ellipse given by the equation $x^2 + xy + y^2 = 3$.

(a) Use implicit differentiation to find $\frac{dy}{dx}$.

(b) Find an equation of the tangent line to the ellipse at the point $(1, 1)$.

(a) $$\frac{d}{dx}\left(x^2+xy+y^2\right) = \frac{d}{dx}(3)$$

$$2x + \left(x\frac{dy}{dx} + y\right) + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}\left(x+2y\right) = -2x-y$$

$$\boxed{\frac{dy}{dx} = \frac{-2x-y}{x+2y}}$$

(b) at $(1,1)$, $\frac{dy}{dx} = \frac{-2-1}{1+2} = -1$.

So tang. line is

$$\boxed{y-1 = -1(x-1)}$$

2. Calculate the following items:

(a) $\frac{dy}{dx}$ if $y = \sqrt{x}\,\sin(x)$

$$\frac{dy}{dx} = \sqrt{x}\cos x + \frac{1}{2\sqrt{x}}\sin x$$

(b) $\frac{d}{d\theta}(\tan(\theta) - 3\cos(\theta))$

$$\sec^2\theta + 3\sin\theta$$

(c) $f^{(2)}(x)$ if $f(x) = x^2 \sin x$

$$f'(x) = x^2\cos x + 2x\sin x$$

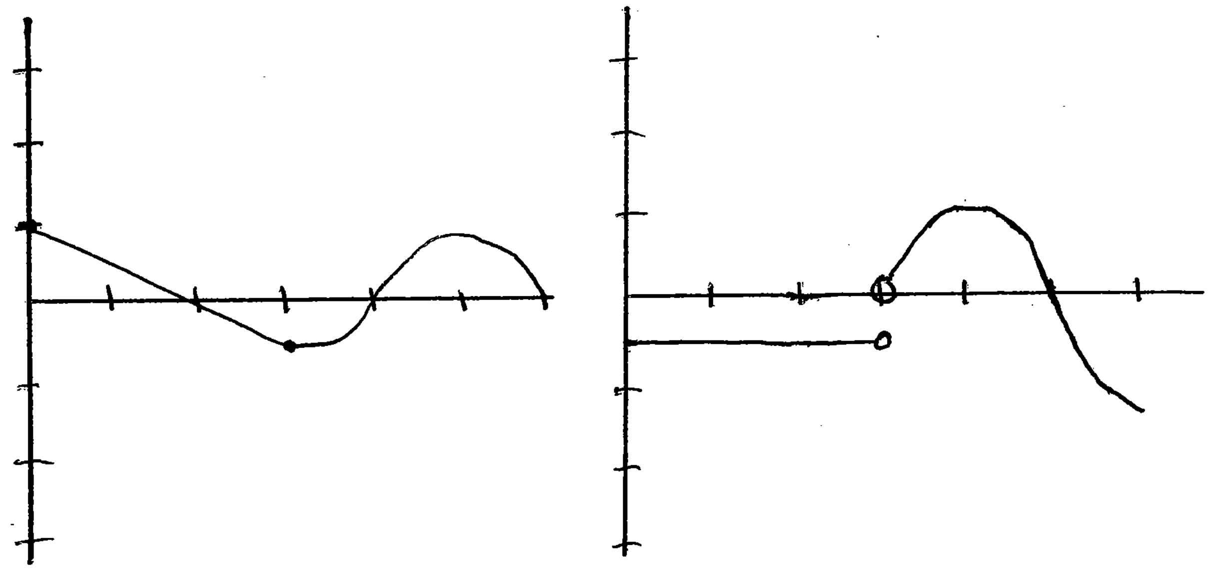
$$f''(x) = -x^2\sin x + 2x\cos x + 2x\cos x + 2\sin x$$

$$= -x^2\sin x + 4x\cos x + 2\sin x$$

(d) $\frac{dy}{dt}$ if $y = \frac{t}{1-t^3}$

$$\frac{dy}{dt} = \frac{(1-t^3) - t(-3t^2)}{(1-t^3)^2}$$

3(a) The first coordinate system below shows the graph of a function $f(x)$. On the second coordinate system, sketch carefully the graph of its derivative $f'(x)$.



3(b) A table of values for f, g, f', and g' is given:

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	4	6
2	1	8	5	7
3	7	2	7	9

(i) If $F(x) = f(f(x))$, find $F'(2)$.
(ii) If $G(x) = g(g(x))$, find $G'(3)$.

$$F'(2) = f'(f(2)) \cdot f'(2)$$
$$= f'(1) \cdot f'(2) = \boxed{20}$$
$$G'(3) = g'(g(3)) \cdot g'(3)$$
$$= g'(2) \cdot g'(3) = \boxed{63}$$

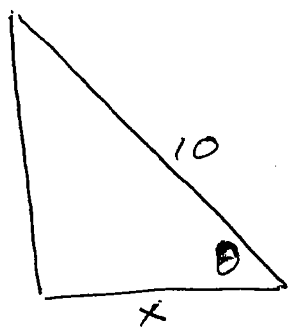
4. A 10 foot ladder is leaning against a wall, and the foot of the ladder slides away from the wall at a rate of one foot per second.

(a) Let x be the distance from the wall to the foot of the ladder, and let θ be the angle between the ladder and the ground. Draw a picture, and write an equation relating x and θ.

(b) Find an equation involving $\frac{dx}{dt}$ and $\frac{d\theta}{dt}$.

(c) How fast is the angle changing when the bottom of the ladder is 6 feet from the wall? Be sure to specify your units.

(a)



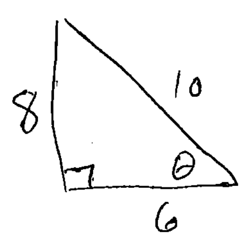
$$\cos\theta = \frac{x}{10}$$

(b)

$$\frac{d}{dt}\left(\cos\theta\right) = \frac{d}{dt}\left(\frac{x}{10}\right)$$

$$-\sin\theta \frac{d\theta}{dt} = \frac{1}{10}\frac{dx}{dt}$$

(c) $x = 6$:



so $\sin\theta = \frac{8}{10}$

also $\frac{dx}{dt} = 1$

$$-\frac{8}{10}\frac{d\theta}{dt} = \frac{1}{10}\cdot 1$$

$$\frac{d\theta}{dt} = \frac{-1}{8} \text{ radians/sec.}$$