

Homework 8

3.6 #40. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\frac{d}{dx} \left(\frac{x^2}{a^2} + \frac{y^2}{b^2} \right) = \frac{d}{dx} (1)$$

$$\frac{2x}{a^2} + \frac{2y}{b^2} \cdot \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{2x}{a^2} \cdot \frac{b^2}{2y} = -\frac{xb^2}{ya^2}$$

At the point (x_0, y_0) , the tangent line is

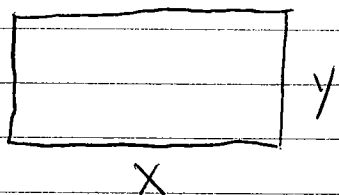
$$y - y_0 = -\frac{x_0 b^2}{y_0 a^2} (x - x_0)$$

$$\frac{y_0}{b^2} (y - y_0) = -\frac{x_0}{a^2} (x - x_0)$$

$$\frac{y_0 y}{b^2} - \frac{y_0^2}{b^2} = -\frac{x_0 x}{a^2} + \frac{x_0^2}{a^2}$$

$$\frac{x_0 x}{a^2} + \frac{y_0 y}{b^2} = \frac{x_0^2}{a^2} + \frac{y_0^2}{b^2} = 1$$

3.8 #4



$$A = xy$$

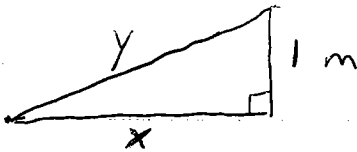
$$\frac{dA}{dt} = \frac{dx}{dt} y + x \frac{dy}{dt}$$

$$x = 20 \text{ cm} \quad \frac{dx}{dt} = 8 \text{ cm/s}$$

$$y = 10 \text{ cm} \quad \frac{dy}{dt} = 3 \text{ cm/s}$$

$$\frac{dA}{dt} = 8 \cdot 10 + 20 \cdot 3 = 140 \text{ cm}^2/\text{s}$$

3.8 #20.



$$x^2 + 1 = y^2$$

$$x = 8 \text{ m}$$

$$y = \sqrt{64 + 1} = \sqrt{65} \text{ m}$$

$$\frac{dy}{dt} = -1 \text{ m/s}$$

$$2x \frac{dx}{dt} = 2y \frac{dy}{dt}$$

$$\frac{dx}{dt} = \frac{y}{x} \frac{dy}{dt}$$

$$\frac{dx}{dt} = -\frac{\sqrt{65}}{8} \text{ m/s}$$

The boat is approaching at

$$\boxed{\frac{\sqrt{65}}{8} \text{ m/s}}$$

3.9 #2 $f(x) = \frac{1}{\sqrt{2+x}}$ $a = 0$

$$L(x) = f(a) + f'(a)(x-a)$$

$$f'(x) = -\frac{1}{2}(2+x)^{-3/2}$$

$$f'(a) = f'(0) = -\frac{1}{2}(2)^{-3/2}$$

$$= -\frac{1}{2\sqrt{8}} = -\frac{1}{4\sqrt{2}}$$

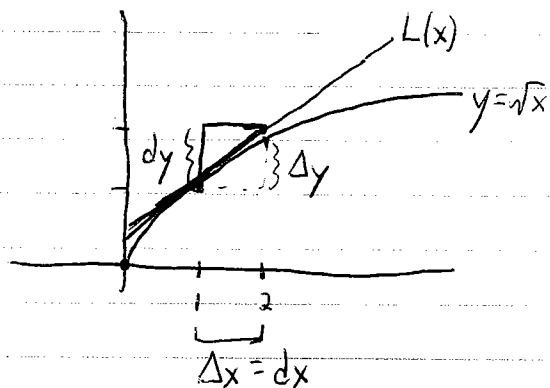
$$f(a) = f(0) = \frac{1}{\sqrt{2}}$$

$$L(x) = \frac{1}{\sqrt{2}} + \left(-\frac{1}{4\sqrt{2}}\right)x = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{8}x$$

#20. $y = \sqrt{x}$, $x=1$, $dx = \Delta x = 1$

$$\Delta y = \sqrt{1+1} - \sqrt{1} = \sqrt{2} - 1$$

$$dy = f'(1)dx = \frac{1}{2\sqrt{1}} \cdot 1 = \frac{1}{2}$$



3.9 #26

Estimate $\frac{1}{1002}$

$$f(x) = \frac{1}{x} \quad f'(x) = -x^{-2} = -\frac{1}{x^2}$$

$$x = 1000, \quad dx = 2$$

$$dy = f'(1000) \cdot 2 = -\frac{2}{1,000,000} = -0.000002$$

$$\begin{aligned} \frac{1}{1002} &\approx \frac{1}{1000} - \frac{2}{1,000,000} = .001 - .000002 \\ &= .000998 \end{aligned}$$