Homework solution 7

3.4 #2. $f(x)=\sqrt{x}\sin x$

$$f'(x)=\sqrt{x}\,(\sin x)'+\sin x\,(\sqrt{x})'$$
$$=\sqrt{x}\cos x+\sin x\cdot\frac{1}{2\sqrt{x}}$$
$$=\sqrt{x}\cos x+\frac{\sin x}{2\sqrt{x}}$$

#4 $y=2\csc x+5\cos x$

$$y'=-2\csc x\cot x-5\sin x$$

#16 $f(x)=x^2\sin x\tan x$

$$f'(x)=x^2\cdot(\sin x\tan x)'+(x^2)'\sin x\tan x$$
$$=x^2(\cos x\tan x+\sin x\cdot\sec^2 x)+2x\sin x\tan x$$
$$=x^2\sin x+x^2\sin x\sec^2 x+2x\sin x\tan x$$

#22 $y=(1+x)\cos x \Rightarrow y'=1\cdot\cos x+(1+x)(-\sin x)$

$$=\cos x-(1+x)\sin x$$

at $(0,1)$ $\quad y'=\cos 0-\sin 0=1$

so the equation of the tagent line is

$y-1=1\cdot(x-0)$ i.e. $y=x+1$

#30 $f(x) = \sec x \Rightarrow f'(x) = \sec x \tan x$

$$f''(x) = \sec x(\sec^2 x) + (\sec x \cdot \tan x) \cdot \tan x$$
$$= \sec x\,(\sec^2 x + \tan^2 x).$$

$$f''(\tfrac{\pi}{4}) = \sec(\tfrac{\pi}{4})[\sec^2(\tfrac{\pi}{4}) + \tan^2(\tfrac{\pi}{4})$$
$$= \sqrt{2}\,((\sqrt{2})^2 + 1)$$
$$= 3\sqrt{2}$$

#32 $g(x) = f(x)\sin x \Rightarrow g'(x) = f'(x) \cdot \sin x + f(x)\cos x$

$$g'(\tfrac{\pi}{3}) = f(\tfrac{\pi}{3})\cos\tfrac{\pi}{3} + \sin\tfrac{\pi}{3} \cdot f'(\tfrac{\pi}{3}) = 4 \cdot \tfrac{1}{2} + \tfrac{\sqrt{3}}{2} \cdot (-2) = 2 - \sqrt{3}$$

#34 $y = \dfrac{\cos x}{2+\sin x} \Rightarrow y' = \dfrac{(2+\sin x)\sin x - \cos x \cdot \cos x}{(2+\sin x)^2}$

$$= \frac{-2\sin x - 1}{(2+\sin x)^2}$$

so if $-2\sin x - 1 = 0$, $y' = 0$ $\quad \sin x = -\frac{1}{2}$ $\quad$ thus

$x = \frac{11\pi}{6} + 2n\pi$ or $x = \frac{7\pi}{6} + 2n\pi$ n is an integer

so $y = \frac{1}{\sqrt{3}}$ or $y = -\frac{1}{\sqrt{3}}$

the points on the curve with horizontal tangents are

$(\frac{11\pi}{6} + 2\pi n \quad \frac{1}{\sqrt{3}}), \ (\frac{7\pi}{6} + 2\pi n, -\frac{1}{\sqrt{3}})$

#40 $\lim_{x\to 0} \frac{\sin 4x}{\sin 6x} = \lim_{x\to 0} \left(\frac{\sin 4x}{x} \cdot \frac{x}{\sin 6x} \right)$

$= \lim_{x\to 0} \frac{4\sin 4x}{4x} \cdot \lim_{x\to 0} \frac{6x}{6\sin 6x}$

$= 4 \cdot \lim_{x\to 0} \frac{\sin 4x}{4x} \cdot \frac{1}{6} \cdot \lim_{x\to 0} \frac{6x}{\sin 6x}$

$= 4 \cdot 1 \cdot \frac{1}{6} \cdot 1$

$= \frac{2}{3}$

3.5

#2 let $u = g(x) = 4 + 3x$ and $y = f(u) = \sqrt{u} = u^{1/2}$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{2} u^{-1/2} \cdot 3 = \frac{3}{2\sqrt{u}} = \frac{3}{2\sqrt{4+3x}}$

#6. $u = g(x) = \sqrt{x}$ $\quad y = f(u) = \sin u$.

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = (\cos u) \cdot \left(\frac{1}{2} x^{-1/2}\right) = \frac{\cos u}{2\sqrt{x}} = \frac{\cos(\sqrt{x})}{2\sqrt{x}}$

#12 $f(t) = \sqrt[3]{1 + \tan t} = (1 + \tan t)^{1/3}$

$f'(t) = \frac{1}{3}(1 + \tan t)^{-2/3} \cdot \sec^2 t$

$= \frac{\sec^2 t}{3\sqrt[3]{(1+\tan t)^2}}$

#23 $y = \sin(x\cos x)$

$$y' = \cos(x\cos x)\cdot\left[\cos x + x(-\sin x)\right]$$

$$= \cos(x\cos x)\cdot(\cos x - x\sin x)$$

#29 $y = \sin(\tan 2x)$

$$y' = \cos(\tan 2x)\cdot(\tan 2x)'$$

$$= \cos(\tan 2x)\cdot \sec^2(2x)\cdot(2x)'$$

$$= \cos(\tan 2x)\cdot\sec^2(2x)\cdot 2$$

$$= 2\cos(\tan 2x)\cdot\sec^2(2x)$$

#52 $y = \sin x + \sin^2 x \Rightarrow y' = \cos x + 2\sin x\cdot\cos x$

at $(0,0)$ $y' = 1$ so the equation of the tangent line

is $y - 0 = 1\cdot(x-0)$ i.e., $y = x$

#63 (a) $h(x) = f(g(x)) \Rightarrow h'(x) = f'(g(x))\cdot g'(x)$ so

$$h'(1) = f'(g(1))\ g'(1) = f'(2)\cdot 6 = 5\cdot 6 = 30$$

(b) $H(x) = g(f(x)) \Rightarrow H'(x) = g'(f(x))\cdot f'(x)$ so

$$H'(1) = g'(f(1))\ f'(1) = g'(3)\cdot 4 = 9\cdot 4 = 36$$

#48 $y = \sin^2(\pi t) = [\sin(\pi t)]^2$

$y' = 2\sin(\pi t)\cdot \cos(\pi t)\cdot \pi = 2\pi \sin\pi t\cdot\cos\pi t = \pi\sin(2\pi t)$

$y'' = \pi\cos(2\pi t)\cdot 2\pi = 2\pi^2\cos(2\pi t)$

#62 $h(x) = \sqrt{4+3f(x)} \Rightarrow h'(x) = \frac{1}{2}(4+3f(x))^{-1/2}\cdot 3\cdot f'(x)$ then

$h'(1) = \frac{1}{2}(4+3f(1))^{-1/2}\cdot 3\cdot f'(1) = \frac{1}{2}(4+3\cdot 7)^{-1/2}\cdot 3\cdot 4 = \frac{6}{\sqrt{25}} = \frac{6}{5}$

3.6

#4 (a) $\frac{d}{dx}(\cos x + \sqrt{y}) = \frac{d}{dx}(5)$

$-\sin x + \frac{1}{2}y^{-1/2}\cdot y' = 0 \Rightarrow y' = 2\sqrt{y}\sin x$

(b) $\cos x + \sqrt{y} = 5 \Rightarrow \sqrt{y} = 5 - \cos x$ so

$y = (5-\cos x)^2$ $\quad y' = 2(5-\cos x)\cdot\sin x$

$= 2\sin x(5-\cos x)$

(c) From part (a) $y' = 2\sqrt{y}\sin x$

$= 2\sqrt{(5-\cos x)^2}\cdot\sin x$

$= 2(5-\cos x)\cdot\sin x$

since $5-\cos x > 0$

#10 $\frac{d}{dx}(y^5 + x^2 y^3) = \frac{d}{dx}(1 + x^4 y)$ so

$$5y^4 y' + x^2 \cdot 3y^2 \cdot y' + y^3 \cdot 2x = 0 + x^4 y' + y \cdot 4x^3$$

$$y'(5y^4 + 3x^2 y^2 - x^4) = 4x^3 y - 2x y^3$$ thus

$$y' = \frac{4x^3 y - 2x y^3}{5y^4 + 3x^2 y^2 - x^4}$$

#14 $\frac{d}{dx}[y \sin(x^2)] = \frac{d}{dx}[x \sin(y^2)]$ so

$$y \cos(x^2) \cdot 2x + \sin(x^2) \cdot y' = x \cos(y^2) \cdot 2y \cdot y' + \sin(y^2) \cdot 1$$

$$y'[\sin(x^2) - 2xy \cos(y^2)] = \sin(y^2) - 2xy \cos(x^2)$$

$$\Rightarrow \quad y' = \frac{\sin(y^2) - 2xy \cos(x^2)}{\sin(x^2) - 2xy \cos(y^2)}$$

#28 $x^{2/3} + y^{2/3} = 4 \Rightarrow \frac{2}{3} x^{-1/3} + \frac{2}{3} y^{-1/3} \cdot y' = 0$

$$\frac{1}{\sqrt[3]{x}} + \frac{y'}{\sqrt[3]{y}} = 0 \Rightarrow y' = -\frac{\sqrt[3]{y}}{\sqrt[3]{x}}$$

if $x = -3\sqrt{3}$, $y = 1$, we have

$$y' = -\frac{1}{(-3\sqrt{3})^{1/3}} = \frac{-(-3\sqrt{3})^{2/3}}{-3\sqrt{3}} = \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}}$$

so an equation of the tangent line is

$$y - 1 = \frac{1}{\sqrt{3}}(x + 3\sqrt{3}) \quad \text{or} \quad y = \frac{1}{\sqrt{3}} x + 4$$