SECTION 1.3

\# 44. Express the function $G(x) = \sqrt[3]{\frac{x}{x+1}}$ in the form of $f \circ g$.

SOLUTION. $g(x) = \frac{x}{x+1}$ and $f(x) = \sqrt[3]{x}$

Verification $f \circ g(x) = f\left(\frac{x}{x+1}\right) = \sqrt[3]{\frac{x}{x+1}}$.

SECTION 2.1

\# 6. We have given,

$$y(t) = 10t - 1.86t^2.$$

At $t = 1$, $y(1) = 10(1) - 1.86(1)^2 = 8.14$

Similarly, at $t = 1+h$,

$$y(1+h) = 10(1+h) - 1.86(1+h)^2$$

$$\therefore V_{ave} = \frac{y(1+h) - y(1)}{(1+h) - 1} = \frac{[10(1+h) - 1.86(1+h)^2] - 8.14}{h} = 6.28 - 1.86h$$

(we assume $h \neq 0$)

$\therefore$ for (i) $[1, 2]$ $h = 1$ $\therefore$ $V_{ave} = 4.42$ m/s.

(ii) $[1, 1.5]$ $h = 0.5$, $V_{ave} = 5.35$ m/s.

(iii) $[1, 1.1]$, $h = 0.1$, $V_{ave} = 6.094$ m/s

(iv) $[1, 1.01]$, $h = 0.01$, $V_{ave} = 6.2614$ m/s. (v) $[1, 1.001]$, $h = 0.001$ $V_{ave} = 6.27814$ m/s.

(b) The instantaneous velocity when $t = 1$ i.e. $h \to 0$ is 6.28 m/s.

SECTION 2.2

6,

(a) $\lim_{x \to -3^-} h(x) = 4$

(b) $\lim_{x \to -3^+} h(x) = 4$

(c) $\lim_{x \to -3} h(x) = 4$

(d) $h(-3)$ is not defined so it does not exist

(e) $\lim_{x \to 0^-} h(x) = 1$

(f) $\lim_{x \to 0^+} h(x) = -1$

(g) $\lim_{x \to 0} h(x)$ does not exist (because $e \neq f$)

(h) $h(0) = 1$

(i) $\lim_{x \to 2^-} h(x) = 2$ and $\lim_{x \to 2^+} h(x) = 2$

$\therefore \lim_{x \to 2} h(x) = 2$

(j) $h(2)$ is not defined (DNE)

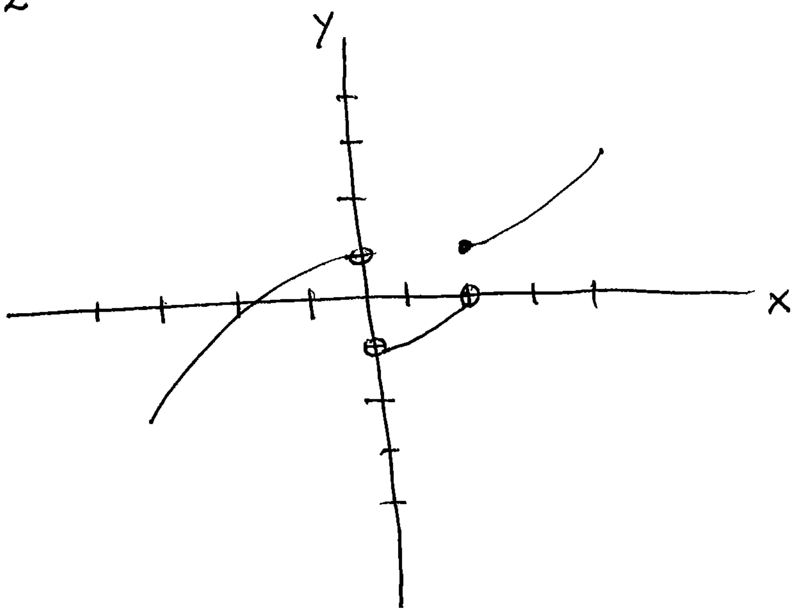
(k) $\lim_{x \to 5^+} h(x) = 3$

(l) $\lim_{x \to 5^-} h(x)$ DNE

(because, $h(x)$ does not approach to any number as x approach to 5 form left).

\# 14. $\lim_{x \to 0^-} f(x) = 1$, $\lim_{x \to 0^+} f(x) = -1$

$\lim_{x \to 2^-} f(x) = 0$, $\lim_{x \to 2^+} f(x) = 1$. $f(2) = 1$ and $f(0)$ undefined.



SECTION 2.3

\# 2, (a) $\lim_{x \to 2} [f(x) + g(x)] = \lim_{x \to 2} f(x) + \lim_{x \to 2} g(x) = 2+0 = 2$

(b) $\lim_{x \to 1} g(x)$ does not exist because $\lim_{x \to 1^+} g(x) \neq \lim_{x \to 1^-} g(x)$

so given limit does not exist.

(c) $\lim_{x \to 0} [f(x) g(x)] = \left(\lim_{x \to 0} f(x)\right) \left(\lim_{x \to 0} g(x)\right)$

$= 0 \cdot 1.3 = 0$

(d) Since, $\lim_{x\to -1} g(x) = 0$ and g is in denominator

but $\lim_{x\to -1} f(x) = -1 \neq 0$ ∴ given limit DNE.

(e) $\lim_{x\to 2} x^3 f(x) = \left(\lim_{x\to 2} x^3\right)\left(\lim_{x\to 2} f(x)\right)$

$= 2^3 \cdot 2 = 16$

(f) $\lim_{x\to 1} \sqrt{3+f(x)} = \sqrt{3+\lim_{x\to 1} f(x)} = \sqrt{3+1} = 2.$

8.

$\lim_{u\to -2} \sqrt{u^4+3u+6}.$

$= \sqrt{\lim_{u\to -2} (u^4+3u+6)}$

$= \sqrt{\lim_{u\to -2} u^4 + \lim_{u\to -2} 3u + \lim_{u\to -2} 6}$

$= \sqrt{(-2)^4 + 3(-2) + 6}$

$= \sqrt{16-6+6}$

$= \sqrt{16}$

$= 4.$