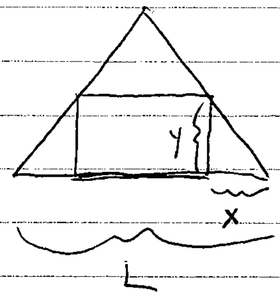
HW #13

4.7 23.



$$A = y(L-2x)$$

$$\tan 60^\circ = \frac{y}{x}$$

$$y = x\tan 60^\circ = \sqrt{3}\,x$$

$$A(x) = \sqrt{3}Lx - 2\sqrt{3}x^2$$

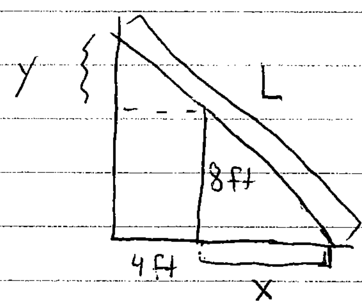
$$A'(x) = \sqrt{3}L - 4\sqrt{3}x = 0$$

$$\Rightarrow x = \frac{L}{4}$$

So, the dimensions which maximize area are $L - 2x = L - \frac{2L}{4} = \boxed{\frac{L}{2}}$

and $y = \sqrt{3}x = \boxed{\frac{\sqrt{3}L}{4}}$

36.



$$L = \sqrt{(x+4)^2 + (y+8)^2}$$

$$\frac{8}{x} = \frac{y+8}{x+4}$$

$$y + 8 = \frac{8x+32}{x}$$

$$y = \frac{32}{x}$$

$$L(x) = \sqrt{(x+4)^2 + \left(\frac{32}{x}+8\right)^2}$$

$$L'(x) = \frac{1}{2}\left((x+4)^2 + \left(\frac{32}{x}+8\right)^2\right)^{-1/2} \cdot \left(2(x+4) + 2\left(\frac{32}{x}+8\right)\left(-\frac{32}{x^2}\right)\right)$$

$$= \frac{x^4 + 4x^3 - 256x - 1024}{x^3\sqrt{(x+4)^2 + \left(\frac{32}{x}+8\right)^2}} = \frac{(x^3 - 256)(x+4)}{x^3\sqrt{(x+4)^2 + \left(\frac{32}{x}+8\right)^2}}$$

$= 0$ when $x = \sqrt[3]{256}$ or -4 (-4 is not in the domain)

$$L(\sqrt[3]{256}) \approx 16.65 \text{ ft}$$

$$A = \pi r \sqrt{r^2+h^2}$$

38. $$27 = V = \frac{1}{3}\pi r^2 h$$

$$h = 81\left(\frac{1}{\pi r^2}\right)$$

$$A(r) = \pi r \sqrt{r^2 + \frac{(81)^2}{\pi^2 r^4}}$$

$$0 = A'(r) = \pi\sqrt{r^2+\frac{(81)^2}{\pi^2 r^4}} + \pi r \cdot \frac{1}{2}\left(r^2+\frac{(81)^2}{\pi^2 r^4}\right)^{-1/2}\left(2r + \frac{-4(81)^2}{\pi^2 r^5}\right)$$

$$-\pi\sqrt{r^2+\frac{81^2}{\pi^2 r^4}} = \pi r\left(r^2+\frac{(81)^2}{\pi^2 r^4}\right)^{-1/2}\left(r - \frac{2(81)^2}{\pi^2 r^5}\right)$$

$$-\left(r^2+\frac{81^2}{\pi^2 r^4}\right) = r\left(r - \frac{2(81)^2}{\pi^2 r^5}\right) = r^2 - \frac{2(81)^2}{\pi^2 r^4}$$

$$\frac{81^2}{\pi^2 r^4} = 2r^2$$

$$\frac{2(81)^2}{\pi^2} = r^6 \qquad r = \sqrt[6]{\frac{2(81)^2}{\pi^2}} \text{ cm}$$

$$h = \frac{81}{\pi} \cdot \sqrt[3]{\frac{\pi^2}{2(81)^2}} \text{ cm}$$

$$S = 6sh - \frac{3}{2}s^2\cot\theta + \left(3s^2\sqrt{3}/2\right)\csc\theta$$

43. (a) $$\frac{dS}{d\theta} = \frac{3}{2}s^2\csc^2\theta - \left(\frac{3s^2\sqrt{3}}{2}\right)\csc\theta\cot\theta$$

(b) $$\frac{dS}{d\theta} = \frac{3}{2}s^2\csc\theta\left(\csc\theta - \sqrt{3}\cot\theta\right) = 0$$

when $\csc\theta = \sqrt{3}\cot\theta$

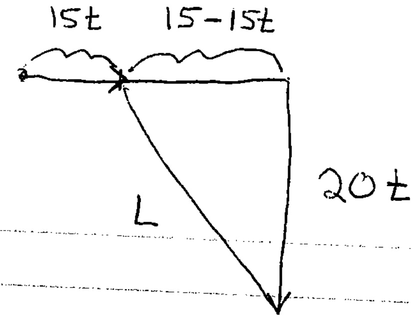
$$\frac{1}{\sin\theta} = \sqrt{3}\cdot\frac{\cos\theta}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sqrt{3}} \qquad \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$\approx 54.7^\circ$$

(c) $$S\left(\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)\right) = 6s\left(h + \frac{s}{2\sqrt{2}}\right)$$

44.



$$L = \sqrt{(20t)^2 + (15(1-t))^2} = \sqrt{400t^2 + 225 - 450t + 225t}$$

$$= \sqrt{625t^2 - 450t + 225}$$

$$L'(t) = \frac{1250t - 450}{2\sqrt{625t^2 - 450t + 225}} = 0 \text{ when}$$

$$1250t = 450$$

$$t = \boxed{\frac{9}{25} \text{ hours}}$$

4.8 2. $x_1 = 9$, $x_2 \approx 6$, $x_3 \approx 7.8$

4. (a) Newton's method fails.

(b) Fails. $f'(1) = 0$

(c) Converges to 2

(d) Fails. $f'(4) = 0$

(e) Converges to 6

8. $f(x) = x^5 + 2$, $x_1 = -1$

$f'(x) = 5x^4$

$$x_2 = -1 - \frac{f(-1)}{f'(-1)} = -1.2$$

$$x_3 = -1.2 - \frac{f(-1.2)}{f'(-1.2)} \approx -1.1529$$

16. $f(x) = x^4 - 2\cos x$

$f'(x) = 4x^3 + 2\sin x$

$x_1 = 1$

$x_2 = 1 - \frac{f(1)}{f'(1)} \approx 1.014184$

$x_3 \approx 1.013958$

$x_4 \approx 1.013958$