section 4.6

#2: $y - x = 100$ so $y = x + 100$

$x \cdot y = x(x+100) = x^2 + 100x$

let $f(x) = x^2 + 100x$ then

$f'(x) = 2x + 100 = 0 \Rightarrow x = -50$

$y = x + 100 = -50 + 100 = 50$

since $f''(x) = 2 > 0$ there is a

absolute minimum of $f(x)$ at $x = -50$

#6. If the rectangle has dimension x and y then its area is

$$x \cdot y = 1000\,m^2$$

so $y = \frac{1000}{x}$

The perimeter $P = 2x + 2y = 2x + 2 \cdot \frac{1000}{x}$

We wish to minimize $P(x) = 2x + \frac{2000}{x}$

for $x > 0$ $P'(x) = 2 - \frac{2000}{x^2} = \frac{2x^2 - 2000}{x^2}$

if $P'(x) = 0$ then $2x^2 = 2000 \Rightarrow x = \sqrt{1000}$

$P''(x) = \frac{4000}{x^3} > 0$ so $P(\sqrt{1000}) = 4\sqrt{1000}$

is an absolute minimum

The dimension of the rectangle with minimal perimeter are

$x = \sqrt{1000} = 10\sqrt{10}$ $\quad y = \frac{1000}{x} = \sqrt{1000} = 10\sqrt{10}$

#12:

Let b be the length of the base of the box and h be the height.

The volume is $32000 = b^2 h$ thus

$$h = \frac{32000}{b^2}$$

The surface Area of the open box is

$$S = b^2 + 4bh = b^2 + 4 \cdot \frac{32000}{b^2} \cdot b$$

$$= b^2 + \frac{4 \cdot 32000}{b}$$

$$S'(b) = 2b - \frac{4 \cdot 32000}{b^2}$$

$$= \frac{2b^3 - 4 \cdot 32000}{b^2}$$

$S'(b) = 0 \Longrightarrow 2b^3 - 4 \cdot 32000 = 0 \Rightarrow b = \sqrt[3]{64000} = 40$

This gives a absolut minimum since $S'(b) < 0$ if $0 < b < 40$ and $S'(b) > 0$ if $b > 0$

the dimension of the box should be ~~40×40×20~~ 40×40×20