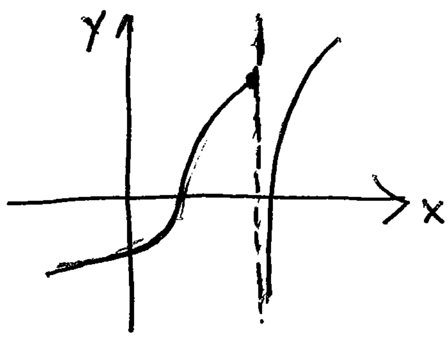
HW #11 (Solution)

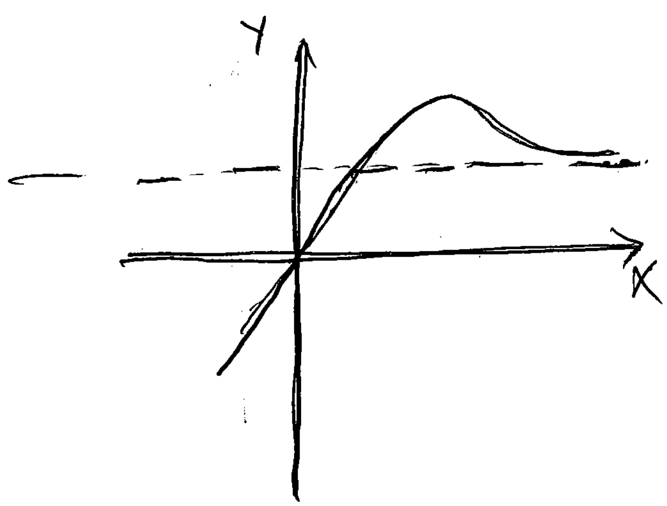
Sec 4.4.

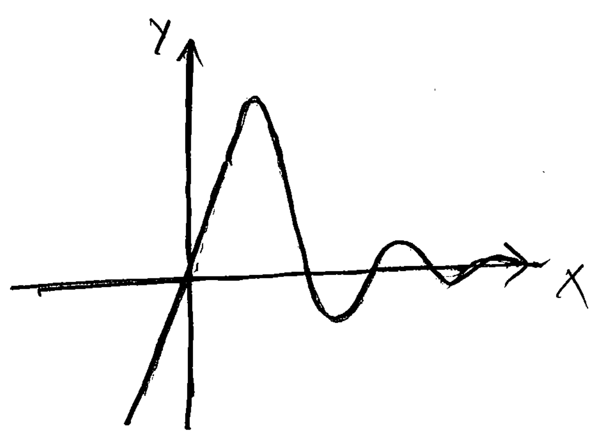
\# 2.

(a) The graph of a function can intersect a vertical asymptote in the sense that it can meet but not cross it.



The graph of a function can intersect a horizontal asymptote. It can even intersects its horizontal asymptote an infinite no of times.

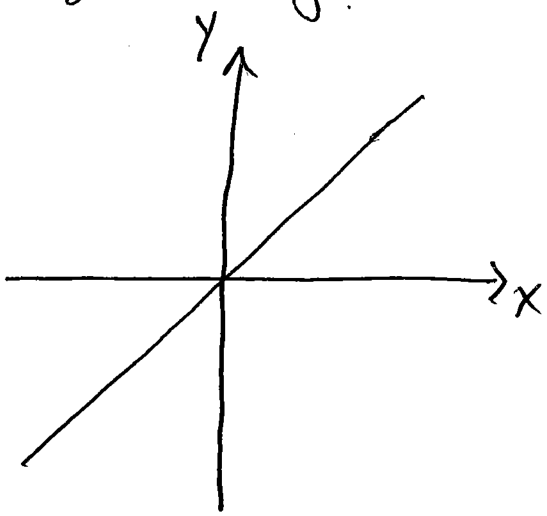


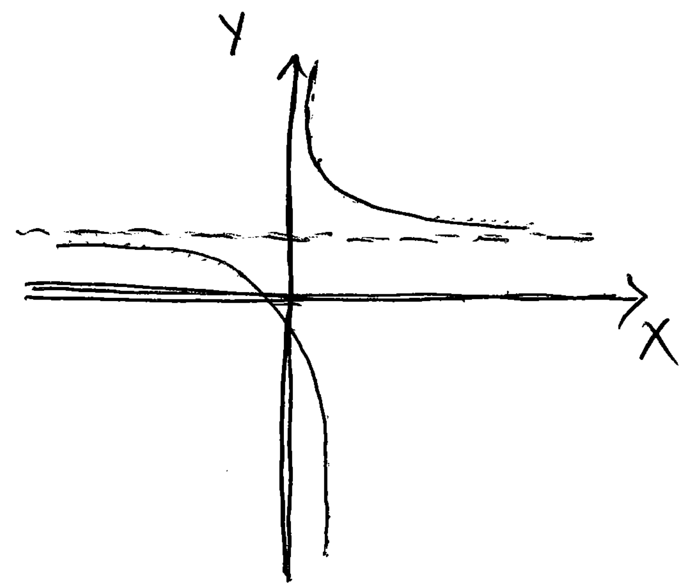


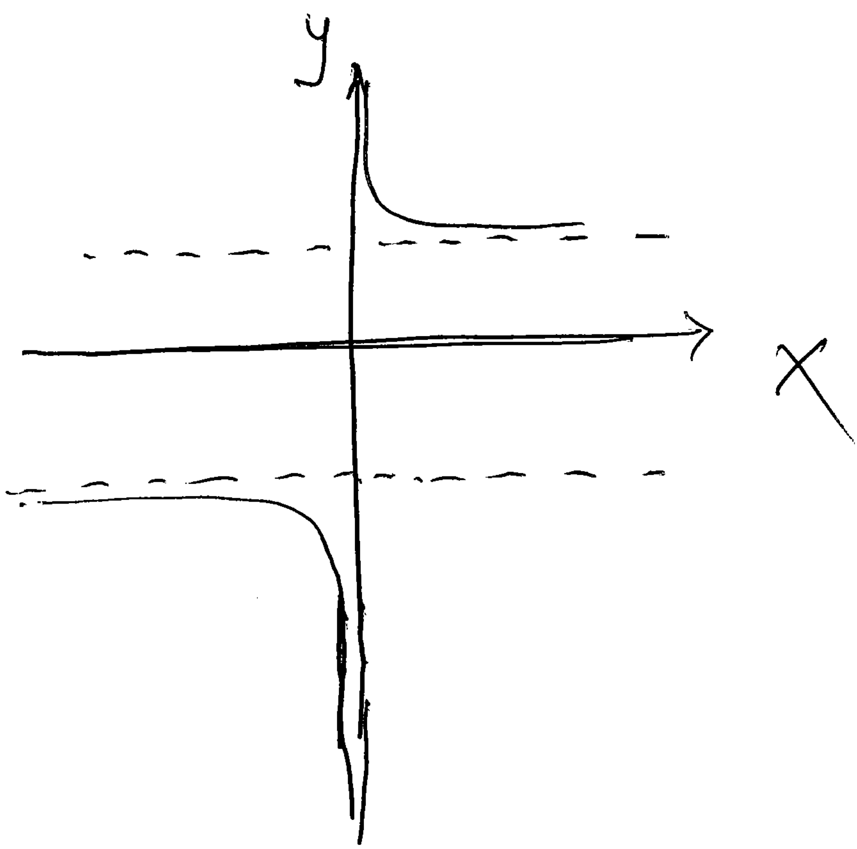
b

The graph of a function can have 0, 1 or 2 horizontal asymptotes.

Eg







#4.

(a) $\lim_{x \to \infty} g(x) = 2$

(b) $\lim_{x \to -\infty} g(x) = -2$

(c) $\lim_{x \to 3} g(x) = \infty$

(d) $\lim_{x \to 0} g(x) = -\infty$

(e) $\lim_{x \to -2^+} g(x) = -\infty$

(f) Vertical $x = -2,\ x = 0,\ x = 3$

& Horizontal $y = -2,\ y = 2$

16. $\lim_{x \to \infty} \dfrac{x+2}{\sqrt{9x^2+1}}$

$$= \lim_{x \to \infty} \frac{1 + 2/x}{\sqrt{9 + 1/x^2}}$$

$$= \frac{1+0}{\sqrt{9+0}}$$

$$= \frac{1}{3}$$

See 4.5

\# 2. $y = x^3 + 6x^2 + 9x$.
$= x(x+3)^2$

(A) $D = \mathbb{R}$
(B) X - interupts are -3 & 0
(C) Y - intercepts ~~are~~ 0
(D) No symmetry (e) No asymptote

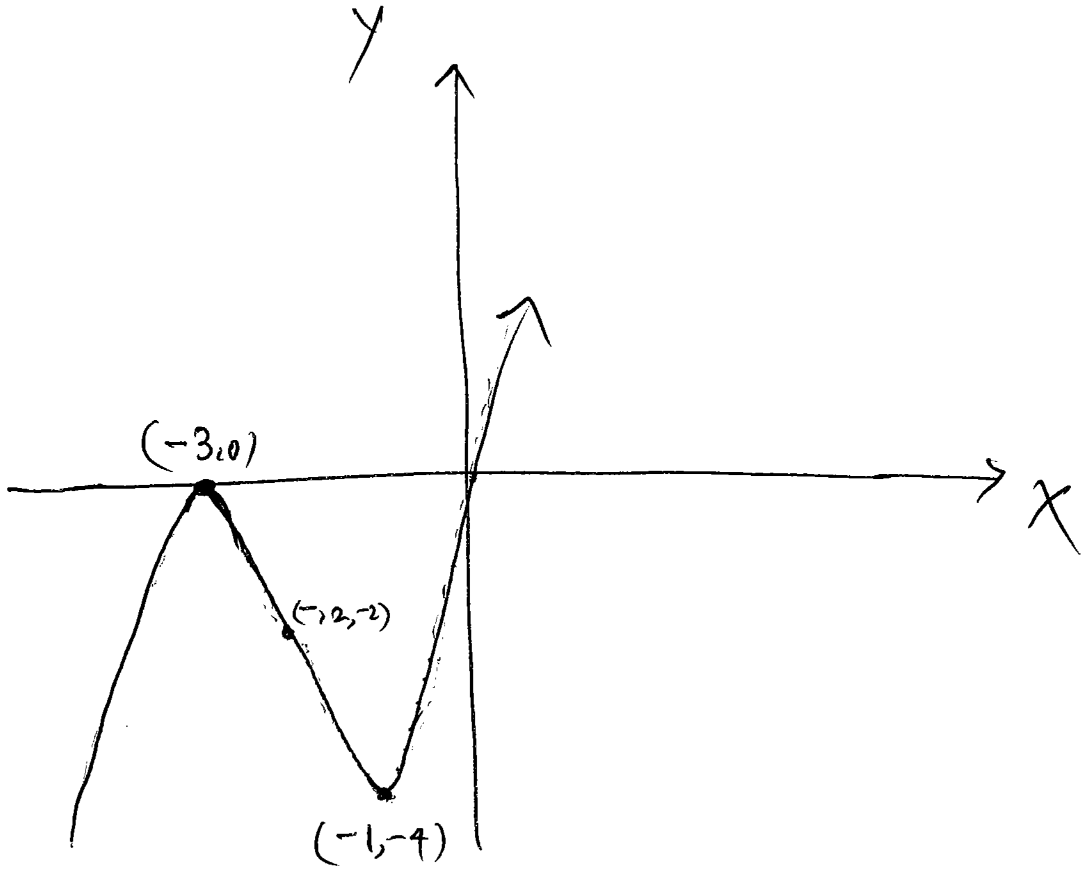
(F) $f'(x) = 3x^2 + 12x + 9$
$= 3(x+1)(x+3) < 0 \iff -3 < x < 1$

$\therefore$ f decreasing on $(-3, -1)$ & increasing on $(-\infty, -3)$ & $(-1, \infty)$

— Loc max value $f(-3) = 0$, Loc min value $f(-1) = -4$

$f''(x) = 6x + 12 = 6(x+2) > 0 \iff x > -2$
$\therefore$ f is CU on $(-2, \infty)$ and CD on $(-\infty, -2)$
IP at $(-2, -2)$.

⑤



14. $f(x) = \dfrac{x^2}{x^2+9}$

$D = \mathbb{R}$

y-interecept $f(0) = 0$

x-interecept $f(x) = 0 \iff x = 0$.

$f(-x) = f(x)$

$\Rightarrow$ f is even functn (symmetric about y axis)

$\lim_{x \to \pm\infty} \left[x^2/x^2+9 \right] = 1 \quad \therefore \boxed{y = 1 \text{ is HA}}$

$\boxed{\text{NO VA}}$

$$f'(x) = \frac{(x^2+9)\,2x - 2x\,x^2}{(x^2+9)^2} = \frac{18x}{(x^2+9)^2} > 0 \Leftrightarrow x > 0$$

$\therefore$ f is increasing on $(0,\infty)$ & decreasing on $(-\infty, 0)$

Loc min value $f(0) = 0$

$\boxed{\text{No Local max}}$

$$f''(x) = \frac{(x^2+9)^2\,18 - 18x \cdot 2(x^2+9)\,2x}{(x^2+9)^{4}}$$

$$= \frac{18(9-3x^2)}{(x^2+9)^3} = \frac{-54(x+\sqrt{3})(x-\sqrt{3})}{(x^2+9)^3} > 0$$

$$\Leftrightarrow -\sqrt{3} < x < \sqrt{3}.$$

$\therefore$ f CU on $(-\sqrt{3}, \sqrt{3})$ & CD on $(-\infty, -\sqrt{3})$ & $(\sqrt{3}, \infty)$

$\therefore$ IPs $(-\sqrt{3}, \frac{1}{4})$ $(3, \frac{1}{4})$.