Homework 10

4.3 #8 (a) f is increasing when $f'(x) > 0$
i.e. on $(2,4)$ and $(6,9)$

(b) f has a local maximum where $f'(x)$ changes from positive to negative: $x=4$

f has a local minimum where $f'(x)$ changes from negative to positive: $x=2$ and $x=6$

(c) f is concave up when f' is increasing
i.e. on $(1,3)$, $(5,7)$, $(8,9)$
f is concave down on $(0,1)$, $(3,5)$, $(7,8)$

(d) f has inflection points where the concavity changes
$x = 1, 3, 5, 7,$ and 8

#16. $f(x) = \frac{x}{x^2+4}$

$$f'(x) = \frac{x^2+4-2x^2}{(x^2+4)^2} = \frac{4-x^2}{(x^2+4)^2}$$

$f'(x) = 0$ when $x = \pm 2$

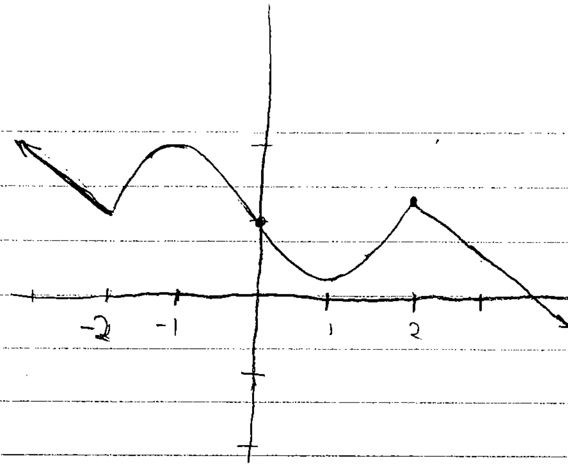
First Derivative Test: When $x < -2$, $f'(x) < 0$
When $-2 < x < 2$, $f'(x) > 0$
When $x > 2$, $f'(x) < 0$

Therefore, f has a local minimum at $x = -2$ and a local maximum at $x = 2$

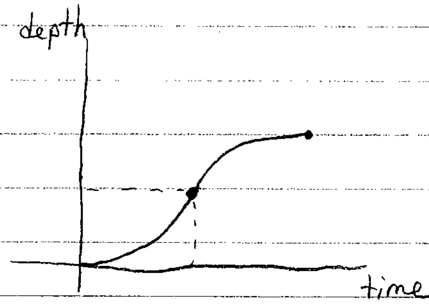
Second Derivative Test: $$f''(x) = \frac{-2x(x^2+4)^2 - 2(x^2+4)\cdot 2x \cdot (4-x^2)}{(x^2+4)^4}$$

$f''(-2) > 0 \Rightarrow$ local minimum $\qquad f''(2) < 0 \Rightarrow$ local maximum

4.3 #22



#52



When the mug is narrower, the depth increases faster. The bottom is wider and the mug gets narrower, so the rate at which the depth increases is itself increasing, making the graph concave up until the time when the ~~t~~ mug is half full, the point of inflection.

4.4 #4. (a) $\lim_{x\to\infty} g(x) = 2$ (b) $\lim_{x\to-\infty} g(x) = -2$

(c) $\lim_{x\to 3} g(x) = \infty$ (d) $\lim_{x\to 0} g(x) = -\infty$

(e) $\lim_{x\to -2^+} g(x) = -\infty$ (f) Vertical asymptotes: $x=-2,\ x=0,\ x=3$

Horizontal asymptotes: $y=2,\ y=-2$

#16. $$\lim_{x\to\infty} \frac{x+2}{\sqrt{9x^2+1}} = \lim_{x\to\infty}\left(\frac{\frac{x+2}{x}}{\frac{\sqrt{9x^2+1}}{x}}\right) = \lim_{x\to\infty}\left(\frac{1+\frac{2}{x}}{\sqrt{9+\frac{1}{x^2}}}\right)$$

$$= \frac{\lim_{x\to\infty}\left(1+\frac{2}{x}\right)}{\lim_{x\to\infty}\sqrt{9+\frac{1}{x^2}}} = \frac{1+0}{\sqrt{\lim_{x\to\infty}\left(9+\frac{1}{x^2}\right)}} = \frac{1}{\sqrt{9+0}} = \boxed{\frac{1}{3}}$$